Beyond slow-roll

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Why?

$$\mathcal{L} = -\frac{1}{2} (\partial \phi)^2 - V(\phi) \quad \rightarrow \quad \mathcal{L} = S(\phi, (\partial \phi)^2, \Box \phi, \ldots)$$

⇒ These corrections must be there. How small?
⇒ Experimentally distinguishable
⇒ DBI inflation: explicit example where hd are important
⇒ What is our guiding principle? What is “natural” in inflation?
⇒ Not so fancy after all:

$$\left[ (\partial \phi)^2 \right]^{\frac{1+w}{2w}}$$

describes a barotropic fluid with $w = p/\rho$
Outline

- DBI as an example
- GWs?
- General approach: EFT for inflation
- Robust equilateral NG (vs local)
- Model dependent GW
- Beyond slow-roll in multifield
Example of an action $\mathcal{L} = P(\phi, X = (\partial \phi)^2)$ where higher derivatives are important.

A probe D3 brane moves towards IR of AdS.

Geometrically there is a speed limit

This limit is encoded in hd operators in DBI action:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{g_s} \sqrt{-g} \left( f(\phi)^{-1} \sqrt{1 + f(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi)} \right)$$

$$f(\phi) \approx \frac{\lambda}{\phi^4} \quad \text{for} \quad \phi \in (\phi_{IR}, \phi_{UV})$$

In the 4d dual, inflaton is moving towards the origin of the moduli space. H.d. operators from integrating out states massless at the origin.

- Does it help with fine tuning?
- Generic?
**DBI predictions**

Reduced speed of sound: \[ c_s^2 \equiv \frac{P_{,X}}{P_{,X} + 2X P_{,XX}} \]

Expanding the actions one gets powers of \( \gamma \gg 1 \)

\[
\mathcal{L}_2 = \frac{1}{g_s} \left[ \frac{1}{2} \gamma^3 \dot{\phi}^2 - \frac{\gamma}{2a^2} (\nabla \phi)^2 + \ldots \right] \quad \mathcal{L}_3 = \frac{1}{g_s} \left[ \frac{\lambda \gamma^5 \dot{\phi}^3}{2\phi^4} - \frac{\lambda \gamma^3 \dot{\phi}}{2a^2 \phi^4} \phi (\nabla \phi)^2 + \ldots \right]
\]

\( c_s = \gamma - 1 < 1 \)

\[
P_S = \frac{1}{8\pi^2 M_P^2} \frac{H^2}{c_s \epsilon} \bigg|_{c_s k = aH}
\]

\[ r = 16 c_s \epsilon \]

- GW are suppressed wrt standard kinetic term \( \otimes \)
- Quite hard to measure both \( r \) and \( f_{\text{NL}}^{\text{equil.}} \) unless \( \epsilon \gg |n_s - 1| \)

(e.g. in exact AdS\(_5\) with \( V = m^2 \phi^2 \), \( |n_s - 1| \) is \( O(\epsilon^2) \))

**NG**

\[
f_{\text{NL}}^{\text{equil.}} = \frac{35}{108} \left( \frac{1}{c_s^2} - 1 \right)
\]

\[-151 < f_{\text{NL}}^{\text{equil.}} < 253 \quad \text{at 95\% C.L.}\]

**WMAP5 limits**
Consistency relation for GWs

• For (and only for) a Lagrangian \( P(\phi, X) = f(\phi)^{-1} \sqrt{1 + f(\phi)X - V(\phi)} \) 

\[
f_{\text{NL}}^{\text{equil.}} = \frac{35}{108} \left( \frac{1}{c_s^2} - 1 \right) \quad r = -8c_s n_t
\]

DBI consistency relation.
It involves GWs + NGs

Probably impossible to measure. Unless \(|n_s - 1| \) is \( O(\epsilon^2) \) with a large \( n_t = -2 \epsilon \)

• A rough verification of the standard consistency relation
(say \( n_t \) is not 10 times larger than expected) gives useful info on \( c_s \)
Lyth in the throat

General conical throat:

\[ ds^2 = h^{-1/2}(y)g_{\mu\nu}dx^\mu dx^\nu + h^{1/2}(y)g_{ij}dy^idy^j \]

\[ g_{ij}dy^idy^j = d\rho^2 + \rho^2 ds^2_{X_5} \]

The inflaton range is limited by the 4d Planck mass:

\[ \left( \frac{\Delta \varphi}{M_P} \right)^2 < \frac{4}{N} \quad N \gg 1 \]

What happens to Lyth’s bound at large speed?

\[ \mathcal{L} = P(\phi, X = (\partial \phi)^2) \quad T_{\mu\nu} = 2P_X \partial_\mu \phi \partial_\nu \phi - P g_{\mu\nu} \]

\[ \epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{XP_{,X}}{M_P^2 H^2} \]

\[ r = 16 c_s \epsilon \quad \frac{\Delta \varphi}{M_P} = \int_0^{N_{\text{end}}} \sqrt{\frac{r}{8 c_s P_{,X}}} dN \]

For DBI: \[ c_s P_{,X} = 1 \]

Same Lyth’s bound. Equally hard to produce GWs...
Lyth in general

\[
\frac{\Delta \varphi}{M_P} = \int_0^{N_{\text{end}}} \sqrt{\frac{r}{8 c_s P_{,X}}} \, dN
\]

⇒ If \( c_s P_{,X} \gg 1 \) I have detectable GWs with sub-Planck displacement.

Do you have a model?

⇒ The bound is obviously not invariant under field redefinition.

Is the correct normalization \( P_{,X}(X = 0, \phi) = 1 \) i.e. a canonical field at low speed?
General approach

with Cheung, Fitzpatrick, Kaplan and Senatore 07

Usual approach to inflation:

1. Take a Lagrangian for a scalar $\mathcal{L}(\phi, \partial_\mu \phi, \square \phi \ldots)$
2. Solve EOM of the scalar + FRW. Find an inflating solution $\ddot{a} > 0$
   \[ \phi = \phi_0(t) \quad ds^2 = -dt^2 + a^2(t) d\bar{x}^2 \]
3. Study perturbations around this solution to work out predictions

We want to focus directly on the theory of perturbations around the inflating solution

- Time diffeomorphisms are broken: $t \rightarrow t + \xi^0(t, \bar{x})$ \quad $\delta \phi \rightarrow \delta \phi + \dot{\phi}_0(t) \xi^0$
- In unitary gauge $\phi(t, \bar{x}) = \phi_0(t)$ the scalar mode is eaten by the graviton:
  3 degrees of freedom. Like in a broken gauge theory.
- The most generic action in unitary gauge
  \[
  S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_{{\text{Pl}}}^2 R + M_{{\text{Pl}}}^2 H g^{00} - M_{{\text{Pl}}}^2 \left(3H^2 + \dot{H}\right) + \frac{M_2(t)^4}{2!} (g^{00} + 1)^2 \right. \\
  \left. + \frac{M_3(t)^4}{3!} (g^{00} + 1)^3 + \ldots - \frac{\overline{M}_2(t)^2}{2} \delta K^\mu{}_{\mu}^2 + \ldots \right].
  \]
Construction of the action in unitary gauge

Inflation. Quasi dS phase with a privileged spatial slicing

Unitary gauge. This slicing coincides with time:
\[ \delta \phi(\vec{x}, t) = 0 \]

Most generic Lagrangian built by metric operators

- Generic functions of time
- \[ \partial_{\mu} t = \delta^0_{\mu} \quad \text{upper 0 indices are ok. E.g. } g^{00} R^{00} \]
- Geometric objects of the 3d spatial slices: e.g. extrinsic curvature \( K_{\mu\nu} \)

\[ S = \int d^4x \sqrt{-g} \left[ F(R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_{\mu}, t) \right] \]

- One can isolate linear terms from the others

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_1^2 R + c(t) g^{00} - \Lambda(t) + \frac{1}{2!} M_2(t)^4 (g^{00} + 1)^2 + \frac{1}{3!} M_3(t)^4 (g^{00} + 1)^3 + \frac{M_1(t)^3}{2} (g^{00} + 1) \delta K^\mu_{\mu} - \frac{M_2(t)^2}{2} \delta K^\mu_{\mu}^2 - \frac{M_3(t)^2}{2} \delta K^\mu_{\nu} \delta K^\nu_{\mu} + \ldots \right],
\]

with \( \delta K_{\mu\nu} = K_{\mu\nu} - \alpha^2 H h_{\mu\nu} \)
Fixing the tadpoles

Background evolution fixes $c(t)$ and $\Lambda(t)$. Higher order terms only affect perturbations

Friedman equations

$$H^2 = \frac{1}{3M_{Pl}^2} [c(t) + \Lambda(t)]$$

give:

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{1}{3M_{Pl}^2} [2c(t) - \Lambda(t)]$$

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_{Pl}^2 R + M_{Pl}^2 \dot{H} g^{00} - M_{Pl}^2 (3H^2 + \dot{H}) + \frac{1}{2!} M_2(t)^4 (g^{00} + 1)^2 + \frac{1}{3!} M_3(t)^4 (g^{00} + 1)^3 + \frac{-\bar{M}_1(t)^3}{2} (g^{00} + 1) \delta K^\mu_{\mu} - \frac{\bar{M}_2(t)^2}{2} \delta K^\mu_{\mu}^2 - \frac{\bar{M}_3(t)^2}{2} \delta K^\mu_{\nu} \delta K^\nu_{\mu} + \ldots \right].$$

Simplest case:

$$\int d^4x \sqrt{-g} \left[ -\frac{1}{2} (\partial \phi)^2 - V(\phi) \right] \rightarrow \int d^4x \sqrt{-g} \left[ -\frac{\dot{\phi}_0(t)^2}{2} g^{00} - V(\phi_0(t)) \right]$$

$$S = \int d^4x \sqrt{-g} P(\dot{\phi}_0(t)^2 g^{00}, \phi(t))$$

$L = P(X, \phi)$, with $X = g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$.

$$M_n^4(t) = \dot{\phi}_0(t)^{2n} \partial^n P / \partial X^n$$
You have a Lagrangian for $P(\phi, X, \phi...)$ with the wanted background and $\phi=t$

- Gets rid of ambiguity of field redefinition

- If I add a e.g. quartic operator this will not affect the previous orders
Slow-roll inflation...

Set to zero all additional operators: \( M_2 = M_3 = M_1 = M_2 \ldots = 0 \)

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_{P1}^2 R - M_{P1}^2 \left( 3H^2(t + \pi) + \dot{H}(t + \pi) \right) + M_{P1}^2 \dot{H}(t + \pi) \left( (1 + \dot{\pi})^2 g^{00} + 2(1 + \dot{\pi})\partial_i\dot{\pi}g^{0i} + g^{ij}\partial_i\dot{\pi}\partial_j\pi \right) \right]
\]

From terms of the form: \( \sim M_{P1}^2 \dot{H}\dot{\pi}\delta g^{00} \) mixing is relevant at \( E_{mix} \sim \epsilon^{1/2} H \)

At \( E \sim H \) + leading order in slow-roll:

\[
S_\pi = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_{P1}^2 R - M_{P1}^2 H \left( \dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2} \right) \right]
\]

\[
\langle \pi_c(\vec{k}_1)\pi_c(\vec{k}_2) \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \frac{H_*^2}{2k_1^3}
\]

After horizon crossing one switch to \( \zeta \) which is (non-linearly) conserved

\[
\dot{\pi} = 0 \quad g_{ij} = a^2(t) \left[ (1 + 2\zeta(t, \vec{x}))\delta_{ij} + \gamma_{ij} \right]
\]

\( t \rightarrow t - \pi(t, \vec{x}) \)

\( \zeta(t, \vec{x}) = -H\pi(t, \vec{x}) \)

Standard results:

\[
\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2) \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \frac{H_*^2}{4\epsilon_* M_{P1}^2 k_1^3} \frac{1}{k_1^3}
\]

\[
n_s - 1 = \frac{d}{d\log k} \log \frac{H_*^4}{|H_*|} = 4H_*^2 - \frac{\dot{H}_*}{H_* H_*}
\]
...and its high energy corrections

Additional operators cannot be really zero. At least radiatively generated

\[ \sim (g^{00} + 1)^2 \dot{H}^2 \log \Lambda \]

Equivalent to an operator

\[ \frac{1}{M_{Pl}^4} (\partial \phi)^4 \log \Lambda \]

The speed of scalar perturbations will be:

\[ 1 - c_s \sim M_2^4 / (|\dot{H}| M_{Pl}^2) \sim |\dot{H}| / M_{Pl}^2 \gtrsim \epsilon^2 \cdot 10^{-10} \]

Not very interesting…

Additional operators may be much bigger with new physics below \( M_p \)

They systematically encode the effect of new physics on slow-roll inflation

\[ \sim \text{Physics beyond SM} \]

Experiments constrain the size of the operators

E.g. GW consistency relation

\[ \langle \gamma^s(\vec{k}_1) \gamma^{s'}(\vec{k}_2) \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \frac{H^2_*}{M_{Pl}^2} \frac{1}{k_1^3} \delta_{ss'} \]

\[ n_g = -2\epsilon_* \]

\[ \langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \frac{1}{c_s^2} \frac{H^2_*}{4\epsilon_* M_{Pl}^2} \frac{1}{k_1^3} \]

\( c_s \) spoils prediction for GW tilt

Rough verification of the relation would set a limit:

\[ M_2^4 \lesssim M_{Pl}^2 |\dot{H}| \]
Small speed of sound…

\[ S_\pi = \int d^4 x \sqrt{-g} \left[ \frac{1}{2} M_{Pl}^2 R - M_{Pl}^2 \dot{H} \left( \dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) + 2 M_2^4 \left( \ddot{\pi} - \frac{\pi (\partial_i \pi)^2}{a^2} \right) - \frac{4}{3} M_3^4 \dddot{\pi} + \ldots \right] \]

Fixed by background!
Pathologies for \( \dot{H} > 0 \)? Not always…
(with Luty, Nicolis and Senatore 06)

\[ c_s^{-2} = 1 - \frac{2 M_2^4}{M_{Pl}^2 \dot{H}} \]

- Lorentz invariance is broken and \( c_s = 1 \) is not protected
- \( c_s > 1 \) not compatible with Lorentz invariant UV theory
(Arkani-Hamed et al 06)

\[ M_2^4 > 0 \]

\[ S_\pi = \int d^4 x \sqrt{-g} \left[ - \frac{M_{Pl}^2 \dot{H}}{c_s^2} \left( \dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} \right) + M_{Pl}^2 \dot{H} \left( 1 - \frac{1}{c_s^2} \right) \left( \ddot{\pi}^3 - \frac{\dddot{\pi} (\partial_i \pi)^2}{a^2} \right) - \frac{4}{3} M_3^4 \dddot{\pi}^3 \ldots \right] \]

As we did in the simplest slow-roll case:

\[ \langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \frac{1}{c_s^*} \frac{H^2_*}{4 \epsilon_* M_{Pl}^2} \frac{1}{k_1^3} \]

\[ n_s = \frac{d}{d \log k} \log \frac{H^4_*}{\dot{H}_* c_s^*} = 4 \frac{\dot{H}_*}{H^2_*} - \frac{\dddot{H}_*}{H_* H_*} - \frac{\dddot{c}_s^*}{c^*_s H_*} \]
...and large Non-Gaussianities

Cubic terms for the Goldstone:

$$M_{\text{Pl}}^2 \dot{H} \left( 1 - \frac{1}{c_s^2} \right) \left( \dot{\pi}^3 - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{4}{3} M_3^4 \dot{\pi}^3$$

- Non-linear realization of diff forces relation between $c_s$ and NG
- Number of independent operators
- Experimentally they give equilaterial NG with slightly different shape
(see Chen, Huang, Kachru and Shiu 06)

Level of non-Gaussianities:

$$\frac{\mathcal{L}_{\dot{\pi} (\nabla \pi)^2}}{\mathcal{L}_2} \sim \frac{H \pi \left( \frac{H}{c_s} \pi \right)^2}{H^2 \pi^2} \sim \frac{H}{c_s^2} \pi \sim \frac{1}{c_s^2} \zeta \quad f_{\text{NL}}^{\text{equil.}} \pi (\nabla \pi)^2 \sim \frac{1}{c_s^2}$$

Experiments set limits on $M_2$
or equivalently on $c_s$

Explicit calculation gives:

$$f_{\text{NL}}^{\text{equil.}} = \frac{85}{324} \cdot \frac{1}{c_s^2}$$
WMAP5 limits: \(-151 < f_{\text{NL}}^{\text{equil.}} < 253\) at 95\% C.L. \(\rightarrow c_s > 0.028\) 
(barring cancellations with M_3)

Planck: 
\[ |f_{\text{NL}}^{\text{equil.}}| < 20 \quad c_s > 0.1 \]

LSS seems promising for local shape, not for this

Can CMBPOL help? Very marginally

Similarly for 4-point function. At leading order in slow-roll: \((g^{00}+1)^2, (g^{00}+1)^3, (g^{00}+1)^4\)

\[ \frac{\mathcal{L}_{(\nabla \pi)^4}}{\mathcal{L}_2} \sim \left(\frac{H}{c_s \pi}\right)^4 \sim \frac{H^2}{c_s^4} \pi^2 \sim \frac{1}{c_s^4} \zeta^2. \]

Contribution linked to \(c_s\):

Huang, Shiu 06
Local VS Equilateral

- The NG signal is concentrated on different configurations.
- They can be easily distinguished (once NG is detected!)

Multi-field models

Modified Lagrangian
Local/equilateral + consistency relation

Under the usual “adiabatic” assumption (a single field is relevant), independently of the inflaton Lagrangian

\[
\lim_{k_1 \to 0} \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = -(2\pi)^3 \delta^3 (\sum_i k_i) P_{k_1} P_{k_3} \left[ \frac{d \log (k_3^3 P_{k_3})}{d \log k_3} + \mathcal{O}(\frac{k_1}{k_3}) \right]
\]

\[
d s^2 = -dt^2 + e^{2\zeta(x)} a^2(t) dx_i dx^i
\]

The long wavelength mode is a frozen background for the other two: it redefines spatial coordinates.

\[n_s - 1 \ll 1\]

In the squeezed limit the 3pf is small and probably undetectable

- Models with a second field have a large 3pf in this limit.

Violation of this relation is a clear, model independent evidence for a second field (same implications as detecting isocurvature).

- This is experimentally achievable if NG is detected.
Bimodal?

- For GWs
  
  \[ r \sim 0.01 \] separates qualitatively different models \[ \Delta \phi \gtrsim M_P \]

  This threshold is also the \sim experimental sensitivity

- For NG

  1. \[ f_{\text{NL}}^{\text{equil.}} \] \ (> \text{few}) separates models with \( c_s \) significantly \(<\ 1\)

  2. \[ f_{\text{NL}}^{\text{local.}} \] \ (> \text{few}) is typical of curvaton/variable decay models

  This threshold is also the \sim experimental sensitivity!!!
**dS limit: ghost inflation**

Arkani-Hamed et al. 03

In the dS limit one has to consider higher derivative terms:

\[ \int d^4 x \sqrt{-g} \left( -\frac{M_2(t)^2}{2} \delta K^\mu_\mu \right) \rightarrow \int d^4 x \sqrt{-g} \left[ -\frac{\tilde{M}^2}{2} \frac{1}{a^4} (\partial_i^2 \pi)^2 \right] \]

Non-relativistic dispersion relation: \( \omega \propto k^2 \)

\[ \int d^4 x \left[ 2M_2^4 \dot{\pi}^2 - \frac{\tilde{M}^2}{2} \frac{1}{a^4} (\partial_i^2 \pi)^2 \right] \]

\[ P_\zeta^{1/2} \sim \left( \frac{H}{\tilde{M}} \right)^{5/4} \]

- High level of NG: \( \frac{\mathcal{L}_{\pi(\nabla \pi)^2}}{\mathcal{L}_2} \sim \left( \frac{H}{\tilde{M}} \right)^{1/4} \)

- GWs are probably small

- \( n_t = 0 \) and they can be tilted red or blue (!!) adding a potential

Here the \( \pi \) language is mandatory!
Beyond slow-roll in multi-field models

\[ P = P \left( X^{IJ}, \phi^K \right) \quad X^{IJ} \equiv -\frac{1}{2} \partial_\mu \phi^I \partial^\mu \phi^J \]

Langlois et al 2008

What is the speed of sound of adiabatic and isocurvature modes?

- If we have only \( P(X = G_{IJ}(\phi)X^{IJ}, \phi) \) locally I can make \( G_{IJ} = \delta_{IJ} \)

\[ P((\partial \phi_1)^2 + (\partial \phi_2)^2 + \ldots) \] Only the adiabatic \( c_s \neq 1 \)

- In general there is no symmetry keeping \( c_s = 1 \) for iso. modes

Indeed

\[ (P_{IJ} + 2P_{MJ}, <IK>X^{MK}) \dot{Q}^I \dot{Q}^J - P_{IJ} h^{ij} \partial_i Q^I \partial_j Q^J \]

In general both adiabatic + isocurvature have \( c_s \neq 1 \)
Multi-field DBI

Motion of a probe brane in: \[ ds^2 = h^{-1/2}(y^K) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y^K) G_{IJ}(y^K) dy^I dy^J \]

\[ L = -T_3 h^{-1} \sqrt{-g} \sqrt{\det(\delta^\mu_\nu + h G_{IJ} \partial^\mu \varphi^I \partial_\nu \varphi^J)} \]

\[ D = 1 - 2 f G_{IJ} X^{IJ} + 4 f^2 X_I^{[I} X_J^{J]} - 8 f^3 X_I^{[I} X_J^J X_K^K] + 16 f^4 X_I^{[I} X_J^J X_K^K X_L^L] \]

Not of the special form: we expect \( c_s \neq 1 \) in all directions.
Indeed all direction share the same \( c_s = \gamma^{-1} \)

Just a geometrical effect: propagation in a direction perp gets \( \gamma^{-1} \)
(independently of which branon I look at)
• But normalization of the action is different in different directions: \( Q_s \sim \frac{Q_{ad}}{c_s} \)

• Isocurvature modes are generated for \( m < H/c_s \). Easier than usual!

\[ \mathcal{P}_\mathcal{R} = \mathcal{P}_{\mathcal{R}*}(1 + T_{\mathcal{RS}}^2) \quad \text{Conversion isocurvature --> adiabatic} \]

Consistency relation:

\[ r \equiv \frac{\mathcal{P}_\mathcal{T}}{\mathcal{P}_\mathcal{R}} = 16\epsilon c_s \frac{1}{1 + T_{\mathcal{RS}}^2} \]

NG: equilateral from horizon crossing + local from conversion iso/adi

In DBI. Horizon crossing terms \( \langle Q_{ad} Q_s Q_s \rangle \) same shape as \( \langle Q_{ad} Q_{ad} Q_{ad} \rangle \)

\[ f_{\text{equil}}^\text{NL} = + \frac{35}{108} \frac{1}{c_s^2} \frac{1}{1 + T_{\mathcal{RS}}^2} \quad + \text{local contributions} \]
Executive summary

1. Good motivations to study non-minimal models.

2. Systematic way of encoding deviations from the minimal slow-roll.

3. Equilateral non-Gaussianities are very robust.
   • They can only be produced in this way
   • They must be there
   • Planck will get down to $|f_{NL}^{\text{equil.}}| < 20$, $c_s > 0.1$

4. GWs are more suppressed than in minimal models ($c_s > 1$).
   They can be seen as a way of constraining these models.
Reintroducing the Goldstone

At sufficiently high energy the Goldstone mode decouples.

\[ S = \int d^4x \left( -\frac{1}{4} \text{Tr} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 \text{Tr} A_\mu A^\mu \right) \quad \text{where} \quad A_\mu = A^{a}_\mu T^a. \]

Gauge transformation:

\[ A_\mu \to U A_\mu U^\dagger + \frac{i}{g} U \partial_\mu U^\dagger \equiv \frac{i}{g} U D_\mu U^\dagger. \quad S = \int d^4x \left( -\frac{1}{4} \text{Tr} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \frac{m^2}{g^2} \text{Tr} D_\mu U^\dagger D_\mu U \right). \]

Gauge invariance is “restored” introducing the Goldstones: \[ U = \exp \left[ iT^a \pi^a (t, \vec{x}) \right] \]

Under a gauge trans. \( \Lambda \) we impose: \[ e^{iT^a \pi^a (t, \vec{x})} = \Lambda (t, \vec{x}) \ e^{iT^a \pi^a (t, \vec{x})} \]

Going to canonical normalization: \( \pi_c \equiv m/g \cdot \pi \) \quad \text{Cutoff:} \quad 4\pi m/g

Mixing with transverse component:

\[ \frac{m^2}{g} A^{a}_\mu \partial^\mu \pi^a = m A^{a}_\mu \partial^\mu \pi_c^a \quad \text{Irrelevant for} \quad E \gg m \]

In the window: \( m \ll E \ll 4\pi m/g \) \quad The physics of the Goldstones is perturbative and decoupled from transverse modes
Doing the same for inflation

Consider for example: \[ \int d^4x \sqrt{-g} \left[ A(t) + B(t)g^{00}(x) \right] \]

Time diff: \[ t \rightarrow \tilde{t} = t + \xi^0(x), \quad \tilde{x} \rightarrow \tilde{\tilde{x}} = \tilde{x} \quad g^{00}(x) \rightarrow \tilde{g}^{00}(\tilde{x}(x)) = \frac{\partial \tilde{x}^0(x)}{\partial x^\mu} \frac{\partial \tilde{x}^0(x)}{\partial x^\nu} g^{\mu\nu}(x) \]

We get: \[ \int d^4\tilde{x} \sqrt{-\tilde{g}(\tilde{x})} \left[ A(\tilde{t} - \xi^0(\tilde{x}(\tilde{x}))) + B(\tilde{t} - \xi^0(\tilde{x}(\tilde{x}))) \frac{\partial(\tilde{t} - \xi^0(\tilde{x}(\tilde{x})))}{\partial \tilde{x}^\mu} \frac{\partial(\tilde{t} - \xi^0(\tilde{x}(\tilde{x})))}{\partial \tilde{x}^\nu} \tilde{g}^{\mu\nu}(\tilde{x}) \right] \]

To restore diff invariance we promote \( \xi \) to a field: \( \xi^0(x(\tilde{x})) \rightarrow -\tilde{\pi}(\tilde{x}) \)

The action \[ \int d^4x \sqrt{-g(x)} \left[ A(t + \pi(x)) + B(t + \pi(x)) \frac{\partial(t + \pi(x))}{\partial x^\mu} \frac{\partial(t + \pi(x))}{\partial x^\nu} g^{\mu\nu}(x) \right] \]

is invariant if \( \pi \) transforms non-linearly: \( \pi(x) \rightarrow \tilde{\pi}(\tilde{x}(x)) = \pi(x) - \xi^0(x) \)

Decoupling limit.
At high energy, no mixing with gravity.

\[ S_\pi = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_{Pl}^2 R - M_{Pl}^2 \dot{H} \left( \dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) + 2M_2^4 \left( \dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 + ... \right] \]

Cosmological perturbations probe the theory at \( E \sim H \).