Paolo Creminelli (ICTP, Trieste)

Beyond slow-roll

CMB polarization workshop: Theory and Foregrounds
Fermilab, 23-26 June 2008

Why?

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) \longrightarrow \mathcal{L} = S(\phi, (\partial\phi)^2, \Box\phi, \ldots)$$

$\Rightarrow$ These corrections must be there. How small?

$\Rightarrow$ Experimentally distinguishable

$\Rightarrow$ DBI inflation: explicit example where hd are important

$\Rightarrow$ What is our guiding principle? What is “natural” in inflation?

$\Rightarrow$ Not so fancy after all:

$\left[(\partial\phi)^2\right]^{\frac{1+w}{2w}}$ describes a barotropic fluid with $w=p/\rho$

Outline

- DBI as an example
- GWs?

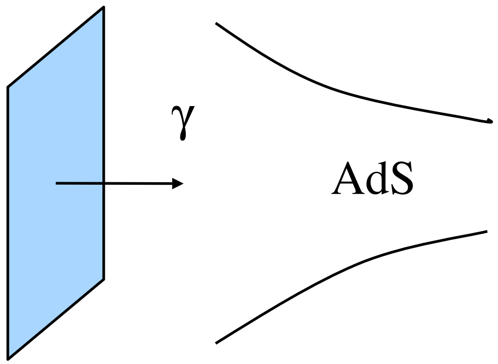
- General approach: EFT for inflation
- Robust equilateral NG (vs local)
- Model dependent GW

- Beyond slow-roll in multifield

DBI inflation

Alishahiha, Silverstein and Tong, 04

Example of an action $\mathcal{L} = P(\phi, X = (\partial\phi)^2)$ where higher derivatives are important



A probe D3 brane moves towards IR of AdS.

Geometrically there is a speed limit

This limit is encoded in hd operators in DBI action:

$$\mathcal{L}_{\text{eff}} = -\frac{1}{g_s}\sqrt{-g}\left(f(\phi)^{-1}\sqrt{1 + f(\phi) g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} + V(\phi)\right)$$

$$f(\phi) \approx \frac{\lambda}{\phi^4} \qquad \text{for } \phi \in (\phi_{IR}, \phi_{UV})$$

In the 4d dual, inflaton is moving towards the origin of the moduli space. H.d. operators from integrating out states massless at the origin.

Conformal invariance

- Does it help with fine tuning?
- Generic?

DBI predictions

Reduced speed of sound: $$c_s^2 \equiv \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

Expanding the actions one gets powers of $\gamma >> 1$

$$\mathcal{L}_2 = \frac{1}{g_s}\left[\frac{1}{2}\gamma^3\dot{\varphi}^2 - \frac{\gamma}{2a^2}(\nabla\varphi)^2 + \ldots\right]$$

$$\mathcal{L}_3 = \frac{1}{g_s}\left[\frac{\lambda\gamma^5\dot{\phi}}{2\phi^4}\dot{\varphi}^3 - \frac{\lambda\gamma^3\dot{\phi}}{2a^2\phi^4}\dot{\varphi}(\nabla\varphi)^2 + \ldots\right]$$

$c_S = \gamma^{-1} < 1$

NG

$$P_S = \frac{1}{8\pi^2 M_P^2}\frac{H^2}{c_s\epsilon}\bigg|_{c_s k = aH}$$

$$f_{\rm NL}^{\rm equil.} = \frac{35}{108}\left(\frac{1}{c_s^2} - 1\right)$$

$$-151 < f_{\rm NL}^{\rm equil.} < 253 \quad \text{at} \;\; 95\% \;\; \text{C.L.}$$

WMAP5 limits

$$r = 16\, c_s\epsilon$$

- GW are suppressed wrt standard kinetic term ☹
- Quite hard to measure both r and $f_{NL}^{equil.}$ unless $\varepsilon >> |n_s-1|$

(e.g. in exact AdS_5 with $V = m^2 \phi^2$, $|n_s-1|$ is $O(\varepsilon^2)$)

Consistency relation for GWs

- For (and only for) a Lagrangian $P(\phi, X) = f(\phi)^{-1}\sqrt{1+f(\phi)X} - V(\phi)$ Lidsey Seery 06

$$f_{\rm NL}^{\rm equil.} = \frac{35}{108}\left(\frac{1}{c_s^2} - 1\right) \qquad r = -8 c_s n_t$$

DBI consistency relation.
It involves GWs + NGs

Probably impossible to measure. Unless $|n_s-1|$ is $O(\varepsilon^2)$ with a large $n_t = -2\,\varepsilon$

- A rough verification of the standard consistency relation (say n_t is not 10 times larger than expected) gives useful info on c_s

Lyth in the throat

Baumann McAllister 06

General conical throat:

$$ds^2 = h^{-1/2}(y) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y) g_{ij} dy^i dy^j \qquad g_{ij} dy^i dy^j = d\rho^2 + \rho^2 ds^2_{X_5}$$

The inflaton range is limited by the 4d Planck mass: $\left(\frac{\Delta\varphi}{M_P}\right)^2 < \frac{4}{N} \qquad N \gg 1$

What happens to Lyth's bound at large speed?

$$\mathcal{L} = P(\phi, X = (\partial\phi)^2) \qquad T_{\mu\nu} = 2P_X \partial_\mu\phi\partial_\nu\phi - P g_{\mu\nu} \qquad \epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{XP_{,X}}{M_P^2 H^2}$$

$$r = 16\, c_s \epsilon \qquad \frac{\Delta\varphi}{M_P} = \int_0^{\mathcal{N}_{\rm end}} \sqrt{\frac{r}{8}\frac{1}{c_s P_{,X}}}\, d\mathcal{N}$$

For DBI: $c_s P_{,X} = 1$

Same Lyth's bound. Equally hard to produce GWs...

Lyth in general

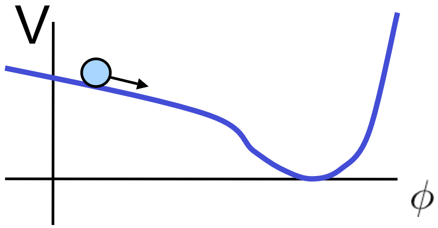
$$\frac{\Delta\varphi}{M_P} = \int_0^{\mathcal{N}_{\rm end}} \sqrt{\frac{r}{8}\frac{1}{c_s P_{,X}}}\,\mathrm{d}\mathcal{N}$$

$\Rightarrow$ If $c_s P_{,X} \gg 1$ I have detectable GWs with sub-Planck displacement.

Do you have a model?

$\Rightarrow$ The bound is obviously not invariant under field redefinition.

Is the correct normalization $P_{,X}(X=0,\phi)=1$ i.e. a canonical field at low speed?



General approach

with Cheung, Fitzpatrick, Kaplan and Senatore 07

Usual approach to inflation:

1. Take a Lagrangian for a scalar $\mathcal{L}(\phi, \partial_\mu \phi, \Box\phi \ldots)$
2. Solve EOM of the scalar + FRW. Find an inflating solution $\ddot{a} > 0$

$$\phi = \phi_0(t) \qquad ds^2 = -dt^2 + a^2(t) d\vec{x}^2$$

3. Study perturbations around this solution to work out predictions

We want to **focus directly on the theory of perturbations** around the inflating solution

- Time diffeomorphisms are broken: $t \to t + \xi^0(t, \vec{x}) \qquad \delta\phi \to \delta\phi + \dot{\phi}_0(t)\xi^0$
- In unitary gauge $\phi(t, \vec{x}) = \phi_0(t)$ the scalar mode is eaten by the graviton:

  3 degrees of freedom. Like in a broken gauge theory.
- The most generic action in unitary gauge

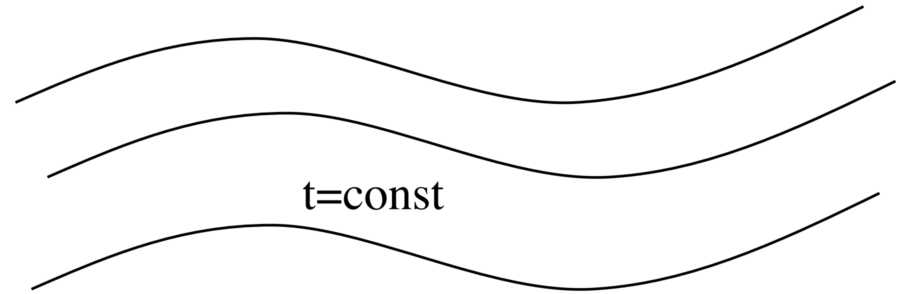
$$S = \int d^4x \, \sqrt{-g} \left[\frac{1}{2} M_{\rm Pl}^2 R + M_{\rm Pl}^2 \dot{H} g^{00} - M_P^2 \left(3H^2 + \dot{H}\right) + \frac{M_2(t)^4}{2!}(g^{00}+1)^2 + \frac{M_3(t)^4}{3!}(g^{00}+1)^3 + \ldots - \frac{\bar{M}_2(t)^2}{2} \delta K^\mu{}_\mu{}^2 + \ldots \right] .$$

Construction of the action in unitary gauge

Inflation. **Quasi dS phase with a privileged spatial slicing**

Unitary gauge. This slicing coincides with time:

$$\delta\phi(\vec{x},t) = 0$$



Most generic Lagrangian built by metric operators

- Generic functions of time
- $\partial_\mu t = \delta^0_\mu$: upper 0 indices are ok. E.g. g^{00} R^{00}
- Geometric objects of the 3d spatial slices: e.g. extrinsic curvature $K_{\mu\nu}$

$$S = \int d^4x \sqrt{-g}\, F(R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu, t)$$

- One can isolate linear terms from the others

$$S = \int d^4x\, \sqrt{-g}\Big[\boxed{\frac{1}{2}M_{\rm Pl}^2 R + c(t)g^{00} - \Lambda(t)} + \frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 + \\ -\frac{\bar{M}_1(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar{M}_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\bar{M}_3(t)^2}{2}\delta K^\mu{}_\nu \delta K^\nu{}_\mu + \ldots\Big],$$

with $\delta K_{\mu\nu} = K_{\mu\nu} - a^2 H h_{\mu\nu}$

Fixing the tadpoles

Background evolution fixes c(t) and Λ(t). Higher order terms only affect perturbations

Friedman equations give:

$$H^2 = \frac{1}{3M_{\rm Pl}^2}\left[c(t)+\Lambda(t)\right]$$

$$\frac{\ddot a}{a} = \dot H + H^2 = -\frac{1}{3M_{\rm Pl}^2}\left[2c(t)-\Lambda(t)\right]$$

$$S = \int d^4x\,\sqrt{-g}\Big[\frac{1}{2}M_{\rm Pl}^2 R + M_{\rm Pl}^2 \dot H g^{00} - M_{\rm Pl}^2(3H^2+\dot H) + \frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 + -\frac{\bar M_1(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar M_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\bar M_3(t)^2}{2}\delta K^\mu{}_\nu \delta K^\nu{}_\mu + \ldots\Big]\,.$$

Simplest case: $\int d^4x\,\sqrt{-g}\left[-\frac{1}{2}(\partial\phi)^2 - V(\phi)\right] \to \int d^4x\,\sqrt{-g}\left[-\frac{\dot\phi_0(t)^2}{2}g^{00} - V(\phi_0(t))\right]$

$$S = \int d^4x\,\sqrt{-g}\;P(\dot\phi_0(t)^2 g^{00},\phi(t))$$

$L = P(X,\phi)$, with $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$.

$$M_n^4(t) = \dot\phi_0(t)^{2n}\partial^n P/\partial X^n$$

Equivalent to the usual language

$$S = \int d^4x\,\sqrt{-g}\Big[\frac{1}{2}M_{\rm Pl}^2 R + M_{\rm Pl}^2 \dot H g^{00} - M_{\rm Pl}^2(3H^2+\dot H) + \frac{1}{2!}M_2(t)^4(g^{00}+1)^2 + \frac{1}{3!}M_3(t)^4(g^{00}+1)^3 + - \frac{\bar M_1(t)^3}{2}(g^{00}+1)\delta K^\mu{}_\mu - \frac{\bar M_2(t)^2}{2}\delta K^\mu{}_\mu{}^2 - \frac{\bar M_3(t)^2}{2}\delta K^\mu{}_\nu \delta K^\nu{}_\mu + \ldots\Big] \,.$$

$$g^{00} \to g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \qquad\qquad t \to \phi$$

- You have a Lagrangian for P(ϕ, X, ϕ...) with the wanted background and ϕ=t
- Gets rid of ambiguity of field redefinition
- If I add a e.g. quartic operator this will not affect the previous orders

Slow-roll inflation…

Set to zero all additional operators: $M_2 = M_3 = \bar{M}_1 = \bar{M}_2 \ldots = 0$

$$S = \int d^4x \, \sqrt{-g} \quad \left[\frac{1}{2} M_{\rm Pl}^2 R - M_{\rm Pl}^2 \left(3H^2(t+\pi) + \dot{H}(t+\pi) \right) + \right.$$
$$\left. + M_{\rm Pl}^2 \dot{H}(t+\pi) \left((1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi}) \partial_i \pi g^{0i} + g^{ij} \partial_i \pi \partial_j \pi \right) \right]$$

From terms of the form: $\sim M_{\rm Pl}^2 \dot{H} \dot{\pi} \delta g^{00}$ mixing is relevant at $E_{\rm mix} \sim \varepsilon^{1/2} H$

At E~H + leading order in slow-roll: $S_\pi = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\rm Pl}^2 R - M_{\rm Pl}^2 \dot{H} \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) \right]$

$$\langle \pi_c(\vec{k}_1) \pi_c(\vec{k}_2) \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \frac{H_*^2}{2k_1^3}$$

A free scalar in dS!

After horizon crossing one switch to ζ which is (non-linearly) conserved

$\pi = 0 \qquad g_{ij} = a^2(t) \left[(1 + 2\zeta(t,\vec{x})) \delta_{ij} + \gamma_{ij} \right]$

$t \to t - \pi(t,\vec{x}) \qquad \zeta(t,\vec{x}) = -H\pi(t,\vec{x})$

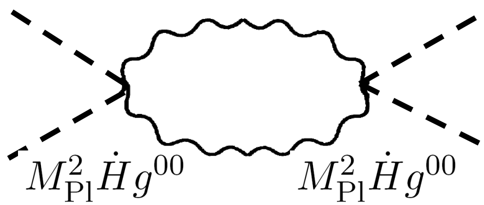
Standard results:

$$\langle \zeta(\vec{k}_1) \zeta(\vec{k}_2) \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) \frac{H_*^2}{4\epsilon_* M_{\rm Pl}^2} \frac{1}{k_1^3}$$

$$n_s - 1 = \frac{d}{d \log k} \log \frac{H_*^4}{|\dot{H}_*|} = 4 \frac{\dot{H}_*}{H_*^2} - \frac{\ddot{H}_*}{H_* \dot{H}_*}$$

…and its high energy corrections

Additional operators cannot be really zero. At least radiatively generated



$$\sim (g^{00}+1)^2 \dot{H}^2 \log\Lambda$$

Equivalent to an operator

$$\frac{1}{M_{\rm Pl}^4}(\partial\phi)^4 \log\Lambda$$

The speed of scalar perturbations will be: $1-c_s \sim M_2^4/(|\dot{H}|M_{\rm Pl}^2) \sim |\dot{H}|/M_{\rm Pl}^2 \gtrsim \epsilon^2 \cdot 10^{-10}$

Not very interesting…

Additional operators may be much bigger with new physics below M_P

They systematically encode the effect of new physics on slow-roll inflation

~ Physics beyond SM

Experiments constrain the size of the operators

E.g. GW consistency relation

$$\langle \gamma^s(\vec{k}_1)\gamma^{s'}(\vec{k}_2)\rangle = (2\pi)^3\delta(\vec{k}_1+\vec{k}_2)\frac{H_*^2}{M_{\rm Pl}^2}\frac{1}{k_1^3}\delta_{ss'} \qquad n_g = -2\epsilon_*$$

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\rangle = (2\pi)^3\delta(\vec{k}_1+\vec{k}_2)\frac{1}{c_{s*}}\cdot\frac{H_*^2}{4\epsilon_* M_{\rm Pl}^2}\frac{1}{k_1^3}$$

c_s spoils prediction for GW tilt

Rough verification of the relation would set a limit: $M_2^4 \lesssim M_{\rm Pl}^2|\dot{H}|$

Small speed of sound…

$$S_\pi = \int d^4x \sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - M_{\rm Pl}^2 \dot H\left(\dot\pi^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + 2M_2^4\left(\dot\pi^2 + \dot\pi^3 - \dot\pi\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot\pi^3 + \ldots\right]$$

Fixed by background!

Pathologies for $\dot H > 0$? Not always…

(with Luty, Nicolis and Senatore 06)

$$c_s^{-2} = 1 - \frac{2M_2^4}{M_{\rm Pl}^2 \dot H}$$

- Lorentz invariance is broken and $c_S=1$ is not protected
- $c_S>1$ not compatible with Lorentz invariant UV theory $\longrightarrow$ $M_2^4 > 0$

(Arkani-Hamed etal 06)

$$S_\pi = \int d^4x\, \sqrt{-g}\left[-\frac{M_{\rm Pl}^2 \dot H}{c_s^2}\left(\dot\pi^2 - c_s^2\frac{(\partial_i\pi)^2}{a^2}\right) + M_{\rm Pl}^2 \dot H\left(1 - \frac{1}{c_s^2}\right)\left(\dot\pi^3 - \dot\pi\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot\pi^3 \ldots\right]$$

As we did in the simplest slow-roll case:

$$\langle\zeta(\vec k_1)\zeta(\vec k_2)\rangle = (2\pi)^3\delta(\vec k_1 + \vec k_2)\frac{1}{c_{s*}}\cdot\frac{H_*^2}{4\epsilon_* M_{\rm Pl}^2}\frac{1}{k_1^3}$$

$$n_s = \frac{d}{d\log k}\log\frac{H_*^4}{|\dot H_*| c_{s*}} = 4\frac{\dot H_*}{H_*^2} - \frac{\ddot H_*}{\dot H_* H_*} - \frac{\dot c_{s*}}{c_{s*} H_*}$$

...and large Non-Gaussianities

Cubic terms for the Goldstone:

$$M_{\rm Pl}^2 \dot H \left(1-\frac{1}{c_s^2}\right)\left(\dot\pi^3 - \dot\pi\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot\pi^3$$

- Non-linear realization of diff forces relation between c_s and NG
- Number of independent operators
- Experimentally they give equilateral NG with slightly different shape

(see Chen, Huang, Kachru and Shiu 06)

Level of non-Gaussianities: $$\frac{\mathcal{L}_{\dot\pi(\nabla\pi)^2}}{\mathcal{L}_2} \sim \frac{H\pi\left(\frac{H}{c_s}\pi\right)^2}{H^2\pi^2} \sim \frac{H}{c_s^2}\pi \sim \frac{1}{c_s^2}\zeta \qquad f_{\rm NL\ \dot\pi(\nabla\pi)^2}^{\rm equil.} \sim \frac{1}{c_s^2}$$

Experiments set limits on M_2
or equivalently on c_s

Explicit calculation gives: $$f_{\rm NL}^{\rm equil.} = \frac{85}{324}\cdot\frac{1}{c_s^2}$$

WMAP5 limits: $-151 < f_{\rm NL}^{\rm equil.} < 253 \quad {\rm at} \;\; 95\% \;\; {\rm C.L.}$ ⟶ $c_s > 0.028$

(barring cancellations with M_3)

Planck: $|f_{\rm NL}^{\rm equil.}| < 20 \qquad c_s > 0.1$

LSS seems promising for local shape, not for this

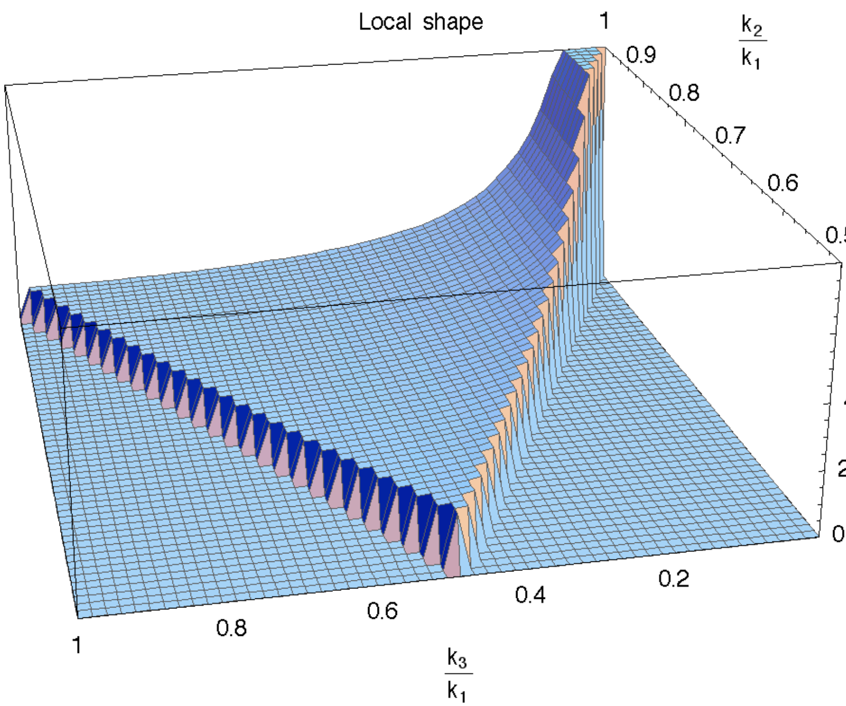
Can CMBPOL help? Very marginally

Similarly for 4-point function. At leading order in slow-roll: $(g^{00}+1)^2$, $(g^{00}+1)^3$, $(g^{00}+1)^4$

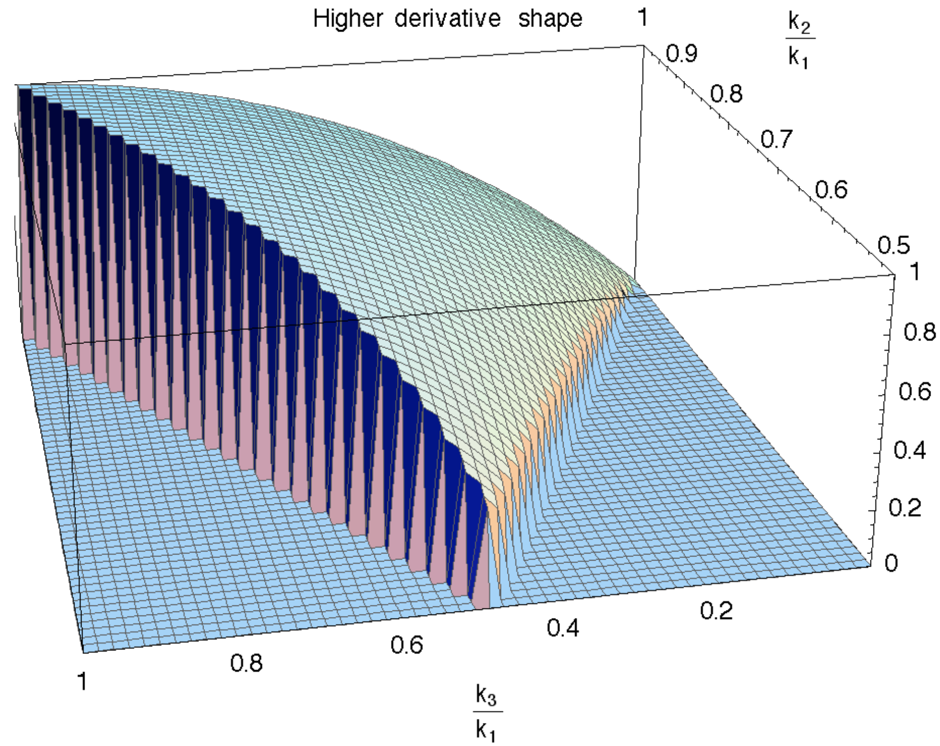
Contribution linked to c_s: $$\frac{\mathcal{L}_{(\nabla\pi)^4}}{\mathcal{L}_2} \sim \frac{\left(\frac{H}{c_s}\pi\right)^4}{H^2\pi^2} \sim \frac{H^2}{c_s^4}\pi^2 \sim \frac{1}{c_s^4}\zeta^2 .$$

Huang, Shiu 06

Local VS Equilateral



Multi-field models



Modified Lagrangian

- The NG signal is concentrated on different configurations.
- They can be easily distinguished (once NG is detected!)

Local/equilateral + consistency relation

Maldacena, 03
PC, Zaldarriaga, 04
Cheung etal, 07
Chen etal, 07

Under the usual "adiabatic" assumption (a single field is relevant), **INDEPENDENTLY** of the inflaton Lagrangian

$$\lim_{k_1\to 0}\langle\zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3}\rangle = -(2\pi)^3\delta^3(\sum_i \vec{k}_i)P_{k_1}P_{k_3}\left[\frac{d\log(k_3^3 P_{k_3})}{d\log k_3} + \mathcal{O}(\frac{k_1}{k_3})\right]$$

$$ds^2 = -dt^2 + e^{2\zeta(x)}a^2(t)dx_i dx^i$$

The long wavelength mode is a frozen background for the other two: it redefines spatial coordinates.

$n_s - 1 \ll 1$ In the squeezed limit the 3pf is small and probably undetectable

- Models with a second field have a large 3pf in this limit.

Violation of this relation is a **clear, model independent evidence** for a second field (same implications as detecting isocurvature).

- This is experimentally achievable if NG is detected.

Bimodal?

- **For GWs**

r ~ 0.01 separates qualitatively different models $\Delta\phi \gtrsim M_P$

This threshold is also the ~ experimental sensitivity

- **For NG**

1. $f_{NL}^{equil.} >$ few separates models with c_s significantly < 1

2. $f_{NL}^{local.} >$ few is typical of curvaton/variable decay models

This threshold is also the ~ experimental sensitivity!!!

dS limit: ghost inflation

Arkani-Hamed etal 03

In the dS limit one has to consider higher derivative terms:

$$\int d^4x\sqrt{-g}\left(-\frac{\bar{M}_2(t)^2}{2}\delta K^\mu{}_\mu{}^2-\frac{\bar{M}_3(t)^2}{2}\delta K^\mu{}_\nu\delta K^\nu{}_\mu\right)\longrightarrow\int d^4x\,\sqrt{-g}\left[-\frac{\bar{M}^2}{2}\frac{1}{a^4}(\partial_i^2\pi)^2\right]$$

$$\bar{M}^2=\bar{M}_2^2+\bar{M}_3^2$$

Non-relativistic dispersion relation: $\omega\propto k^2$

$$\int d^4x\left[2M_2^4\dot{\pi}^2-\frac{\bar{M}^2}{2}\frac{1}{a^4}(\partial_i^2\pi)^2\right]$$

$$P_\zeta^{1/2}\simeq\left(\frac{H}{M}\right)^{5/4}$$

- High level of NG: $\dfrac{\mathcal{L}_{\dot{\pi}(\nabla\pi)^2}}{\mathcal{L}_2}\sim\left(\dfrac{H}{M}\right)^{1/4}$

- GWs are probably small

- $n_t = 0$ and they can be tilted red or blue (!!) adding a potential

Here the π language is mandatory!

Beyond slow-roll in multi-field models

$$P = P\left(X^{IJ}, \phi^K\right) \qquad X^{IJ} \equiv -\frac{1}{2}\partial_\mu\phi^I\partial^\mu\phi^J$$

Langlois etal 2008

What is the speed of sound of adiabatic and isocurvature modes?

- If we have only $P(X = G_{IJ}(\phi)X^{IJ}, \phi)$ locally I can make $G_{IJ} = \delta_{IJ}$

$$P((\partial\phi_1)^2 + (\partial\phi_2)^2 + \ldots)$$

Only the adiabatic $c_s \neq 1$

- In general there is no symmetry keeping $c_s=1$ for iso. modes

Indeed $\left(P_{<IJ>} + \underline{\underline{2P_{<MJ>,<IK>}X^{MK}}}\right)\dot{Q}^I\dot{Q}^J - P_{<IJ>}h^{ij}\partial_i Q^I\partial_j Q^J$

In general both adiabatic + isocurvature have $c_s \neq 1$

Multi-field DBI

Motion of a probe brane in: $$ds^2 = h^{-1/2}(y^K)\, g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y^K)\, G_{IJ}(y^K)\, dy^I dy^J$$

$$L = -T_3 h^{-1}\sqrt{-g}\sqrt{\det(\delta^\mu_\nu + h\, G_{IJ}\partial^\mu\varphi^I\partial_\nu\varphi^J)}$$

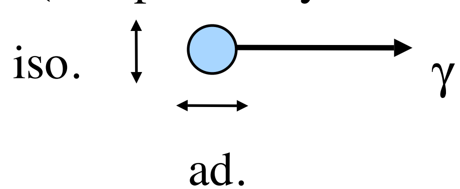
$$\mathcal{D} = 1 - 2fG_{IJ}X^{IJ} + 4f^2 X_I^{[I}X_J^{J]} - 8f^3 X_I^{[I}X_J^{J}X_K^{K]} + 16f^4 X_I^{[I}X_J^{J}X_K^{K}X_L^{L]}$$

Not of the special form: we expect $c_s \neq 1$ in all directions.

Indeed all direction share the **same** $c_s = \gamma^{-1}$

Just a geometrical effect: propagation in a direction perp gets γ^{-1}

(independently of which branon I look at)



- But normalization of the action is different in different directions: $Q_s \simeq \frac{Q_{ad}}{c_s}$

- Isocurvature modes are generated for $m < H/c_s$. Easier than usual!

$$\mathcal{P}_{\mathcal{R}} = \mathcal{P}_{\mathcal{R}*}(1 + T^2_{\mathcal{RS}})$$

Conversion isocurvature --> adiabatic

Consistency relation: $$r \equiv \frac{\mathcal{P}_{\mathcal{T}}}{\mathcal{P}_{\mathcal{R}}} = 16\epsilon c_s \frac{1}{1 + T^2_{\mathcal{RS}}}$$ ☹2

NG: equilateral from horizon crossing + local from conversion iso/adi

In DBI. Horizon crossing terms $\langle Q_{ad} Q_s Q_s \rangle$ same shape as $\langle Q_{ad} Q_{ad} Q_{ad} \rangle$

$$f_{\rm NL}^{\rm equil} = + \frac{35}{108} \frac{1}{c_s^2} \frac{1}{1 + T^2_{\mathcal{RS}}}$$ ☹ + local contributions

Executive summary

1. Good motivations to study non-minimal models.

2. Systematic way of encoding deviations from the minimal slow-roll.

3. Equilateral non-Gaussianities are very robust.
   - They can only be produced in this way
   - They must be there
   - Planck will get down to $|f_{NL}^{equil.}| < 20$, $c_s > 0.1$

4. GWs are more suppressed than in minimal models ($c_s > 1$?).

   They can be seen as a way of constraining these models.

Reintroducing the Goldstone

At sufficiently high energy the Goldstone mode decouples.

$$S = \int d^4x \, -\frac{1}{4}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 \mathrm{Tr}\, A_\mu A^\mu \qquad \text{where } A_\mu = A_\mu^a T^a .$$

Gauge transformation:

$$A_\mu \to U A_\mu U^\dagger + \frac{i}{g} U \partial_\mu U^\dagger \equiv \frac{i}{g} U D_\mu U^\dagger \, . \qquad S = \int d^4x \, -\frac{1}{4}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\frac{m^2}{g^2}\mathrm{Tr} D_\mu U^\dagger D_\mu U \, .$$

Gauge invariance is "restored" introducing the Goldstones: $U = \exp\left[i T^a \pi^a(t,\vec{x}) \right]$

Under a gauge trans. Λ we impose: $e^{iT^a \tilde{\pi}^a(t,\vec{x})} = \Lambda(t,\vec{x})\, e^{iT^a \pi^a(t,\vec{x})}$

Going to canonical normalization: $\pi_c \equiv m/g \cdot \pi$ Cutoff: $4\pi m/g$

Mixing with transverse component: $\frac{m^2}{g} A_\mu^a \partial^\mu \pi^a = m A_\mu^a \partial^\mu \pi_c^a$ Irrelevant for $E \gg m$

In the window: $m \ll E \ll 4\pi m/g$ The physics of the Goldstones is perturbative and decoupled from transverse modes

Doing the same for inflation

Consider for example: $\int d^4x \sqrt{-g}\,[A(t) + B(t) g^{00}(x)]$

Time diff: $t \to \tilde{t} = t + \xi^0(x),\ \vec{x} \to \tilde{\vec{x}} = \vec{x}$ $\qquad g^{00}(x) \to \tilde{g}^{00}(\tilde{x}(x)) = \frac{\partial \tilde{x}^0(x)}{\partial x^\mu} \frac{\partial \tilde{x}^0(x)}{\partial x^\nu} g^{\mu\nu}(x)$

We get:

$$\int d^4\tilde{x} \sqrt{-\tilde{g}(\tilde{x})} \left[A(\tilde{t} - \xi^0(x(\tilde{x}))) + B(\tilde{t} - \xi^0(x(\tilde{x}))) \frac{\partial(\tilde{t} - \xi^0(x(\tilde{x})))}{\partial \tilde{x}^\mu} \frac{\partial(\tilde{t} - \xi^0(x(\tilde{x})))}{\partial \tilde{x}^\nu} \tilde{g}^{\mu\nu}(\tilde{x}) \right]$$

To restore diff invariance we promote ξ to a field: $\xi^0(x(\tilde{x})) \to -\tilde{\pi}(\tilde{x})$

The action

$$\int d^4x \sqrt{-g(x)} \left[A(t + \pi(x)) + B(t + \pi(x)) \frac{\partial(t + \pi(x))}{\partial x^\mu} \frac{\partial(t + \pi(x))}{\partial x^\nu} g^{\mu\nu}(x) \right]$$

is invariant if π transforms non-linearly: $\pi(x) \to \tilde{\pi}(\tilde{x}(x)) = \pi(x) - \xi^0(x)$

Decoupling limit.
At high energy, no mixing with gravity.

Cosmological perturbations probe the theory at E ~ H

$$S_\pi = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\rm Pl}^2 R - M_{\rm Pl}^2 \dot{H} \left(\dot{\pi}^2 - \frac{(\partial_i \pi)^2}{a^2} \right) + 2 M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 + \ldots \right]$$