Foreground removal with Commander

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Institute of Theoretical Astrophysics, Oslo
June 24, 2008

See Eriksen et al. 2008, ApJ, 676, 10 (astro-ph:0709.1058) for full details

Traditional CMB analysis

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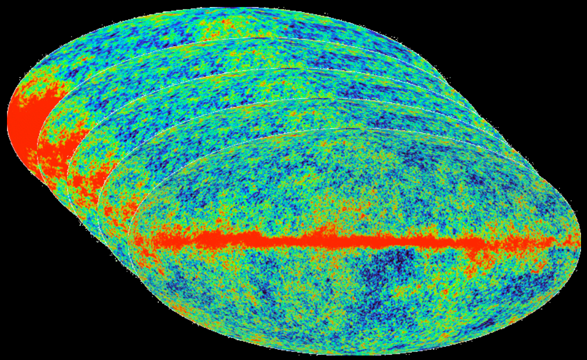

Frequency maps

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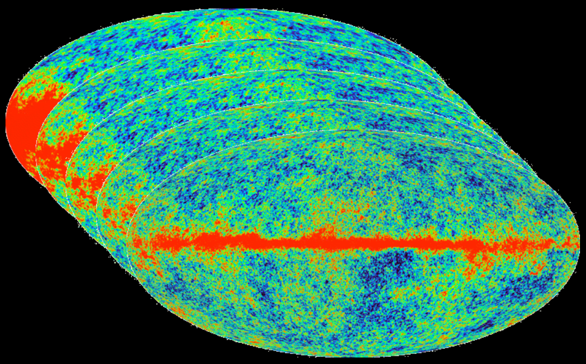

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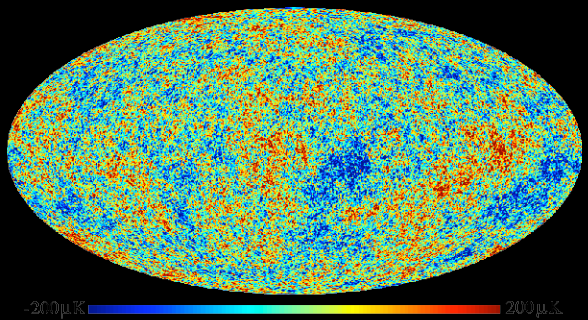

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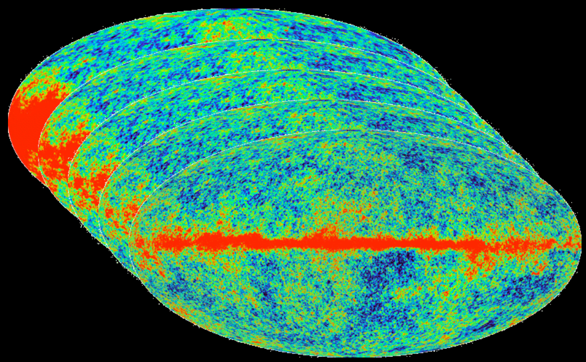

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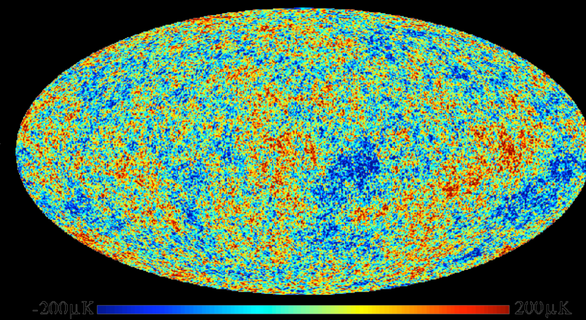

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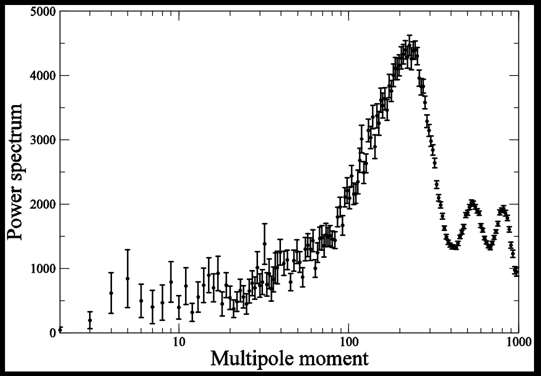
"Generalized FFT"



Power spectrum

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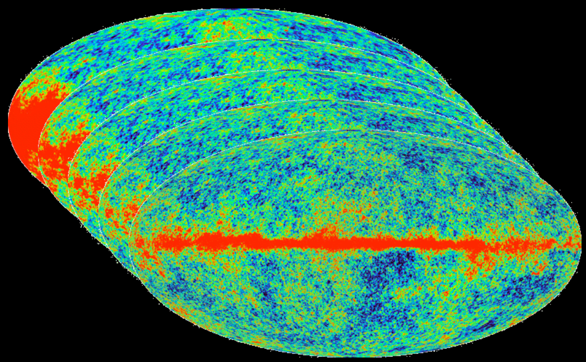

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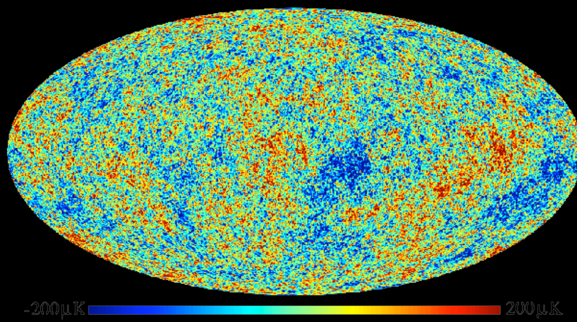

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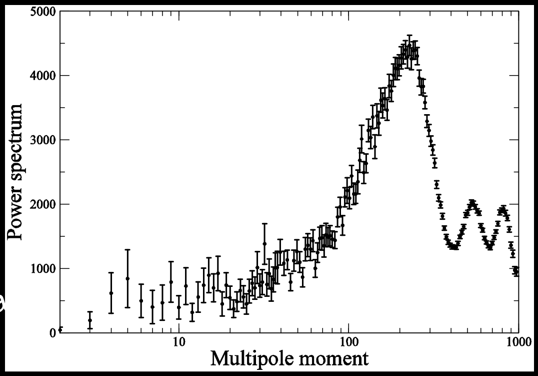
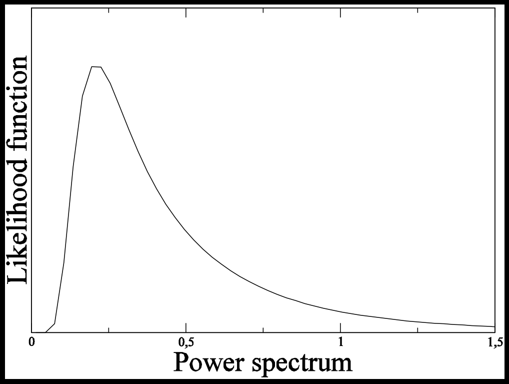


Power spectrum

Likelihood fitting formula



Likelihood

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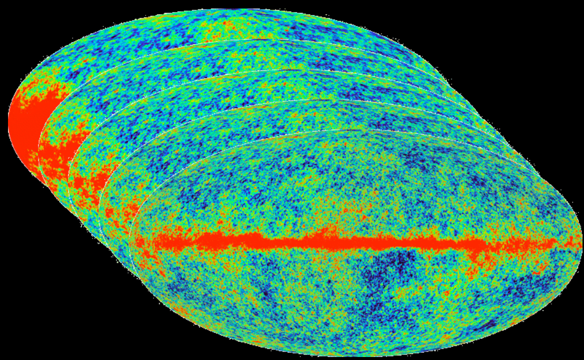

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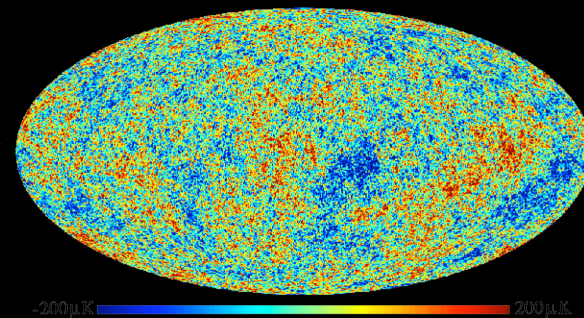

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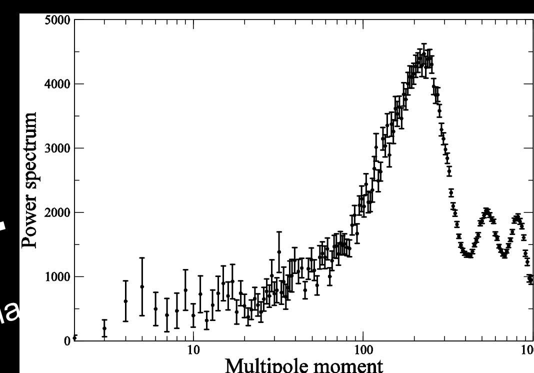

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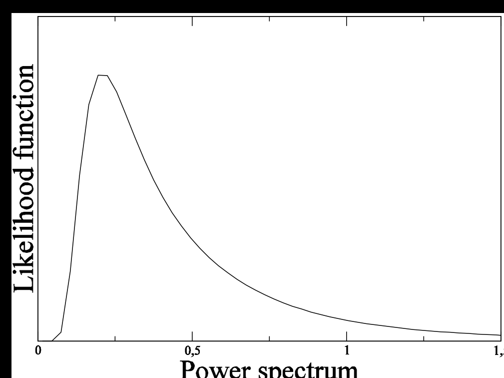


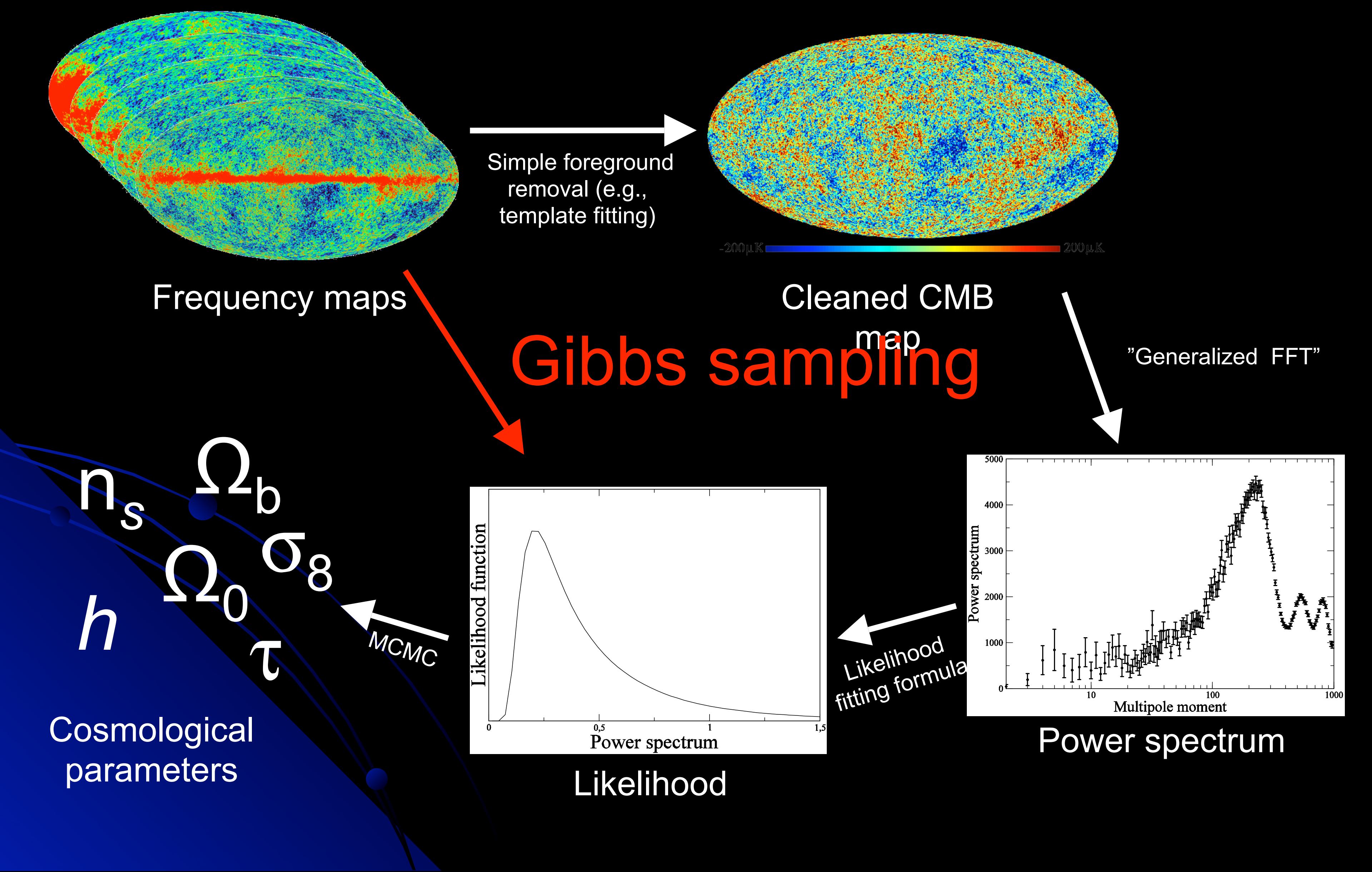
Likelihood

MCMC

n_s Ω_b σ_8 Ω_0 h τ

Cosmological parameters

CMB analysis by Gibbs sampling



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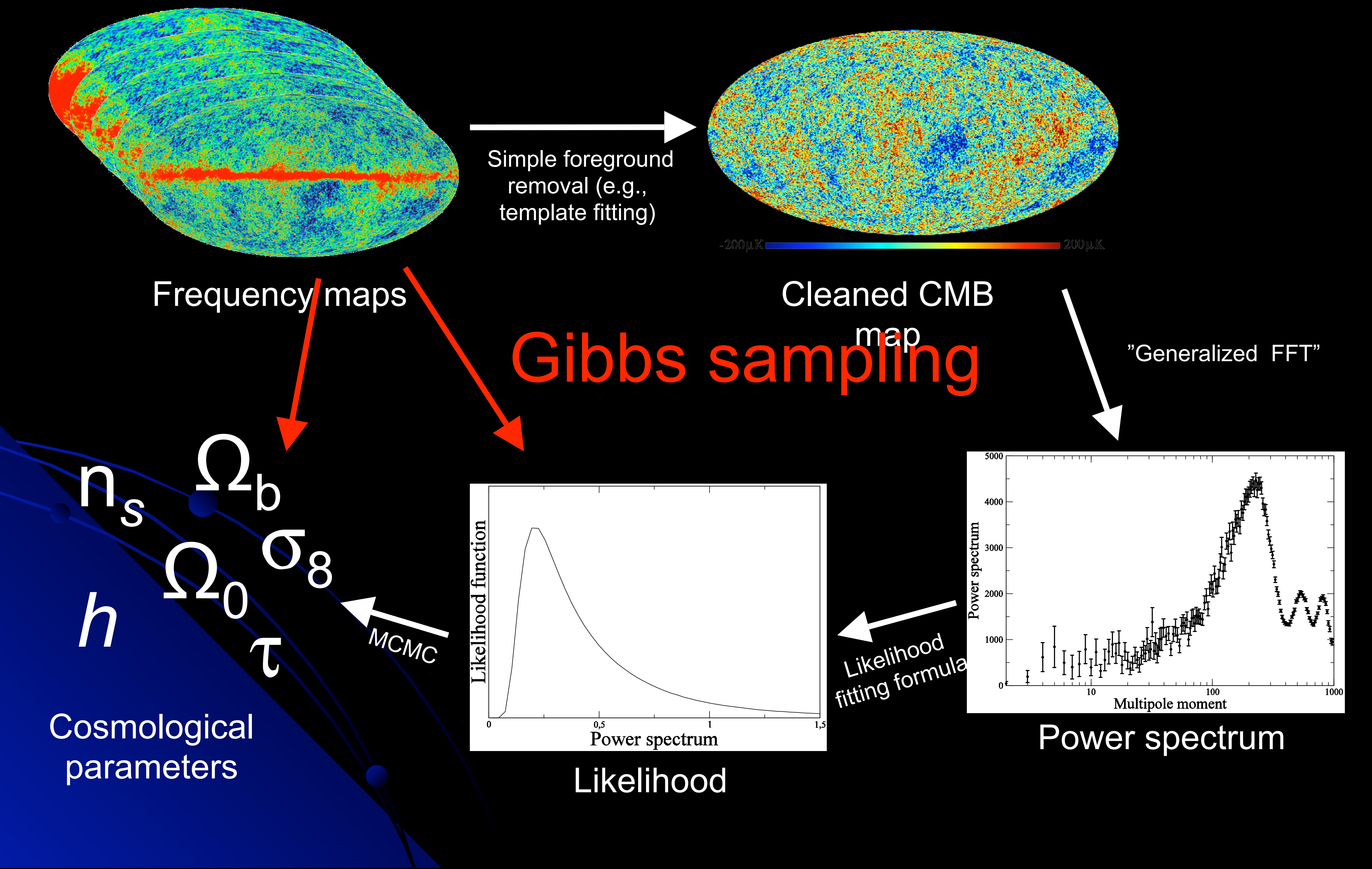

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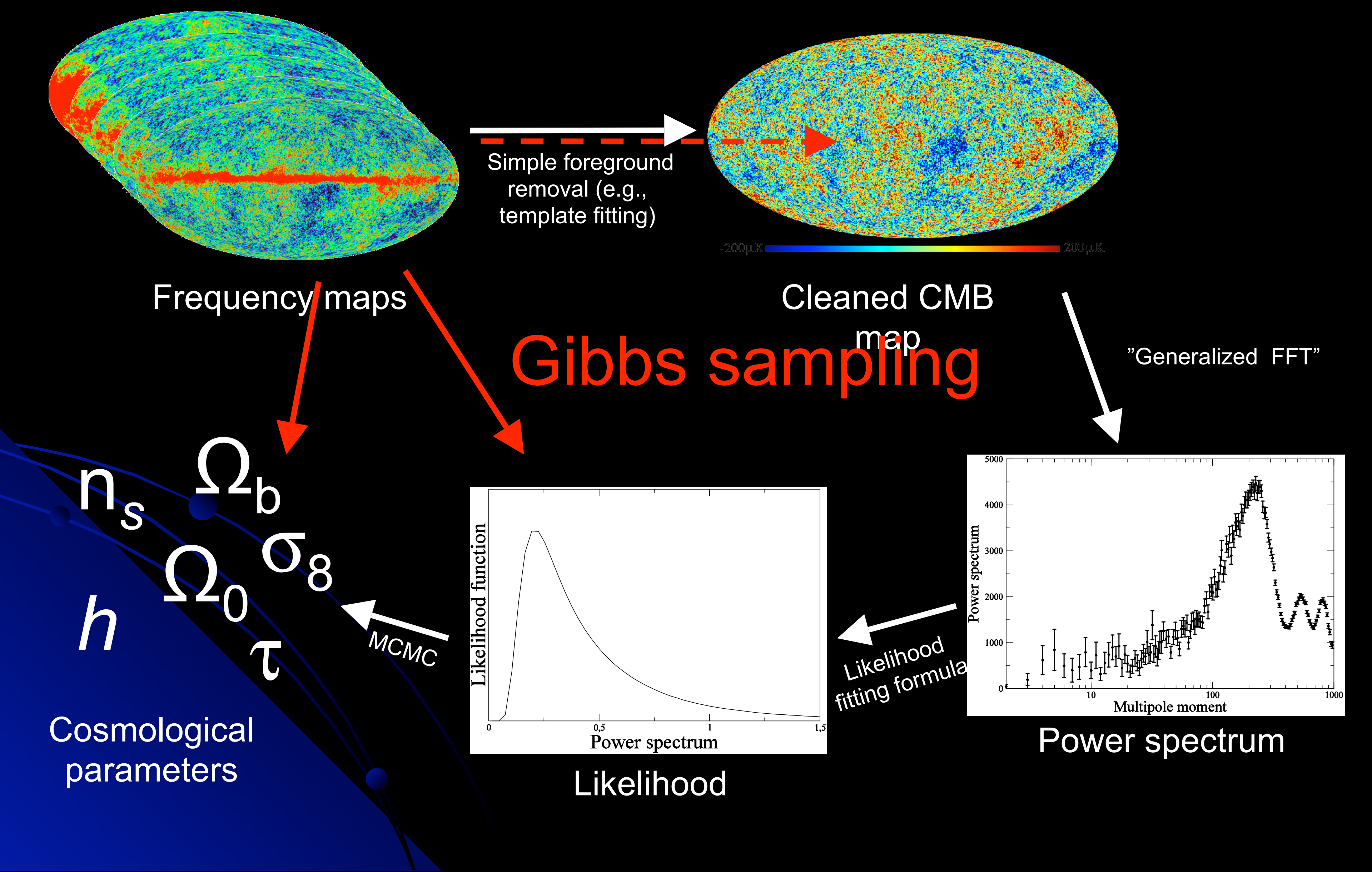

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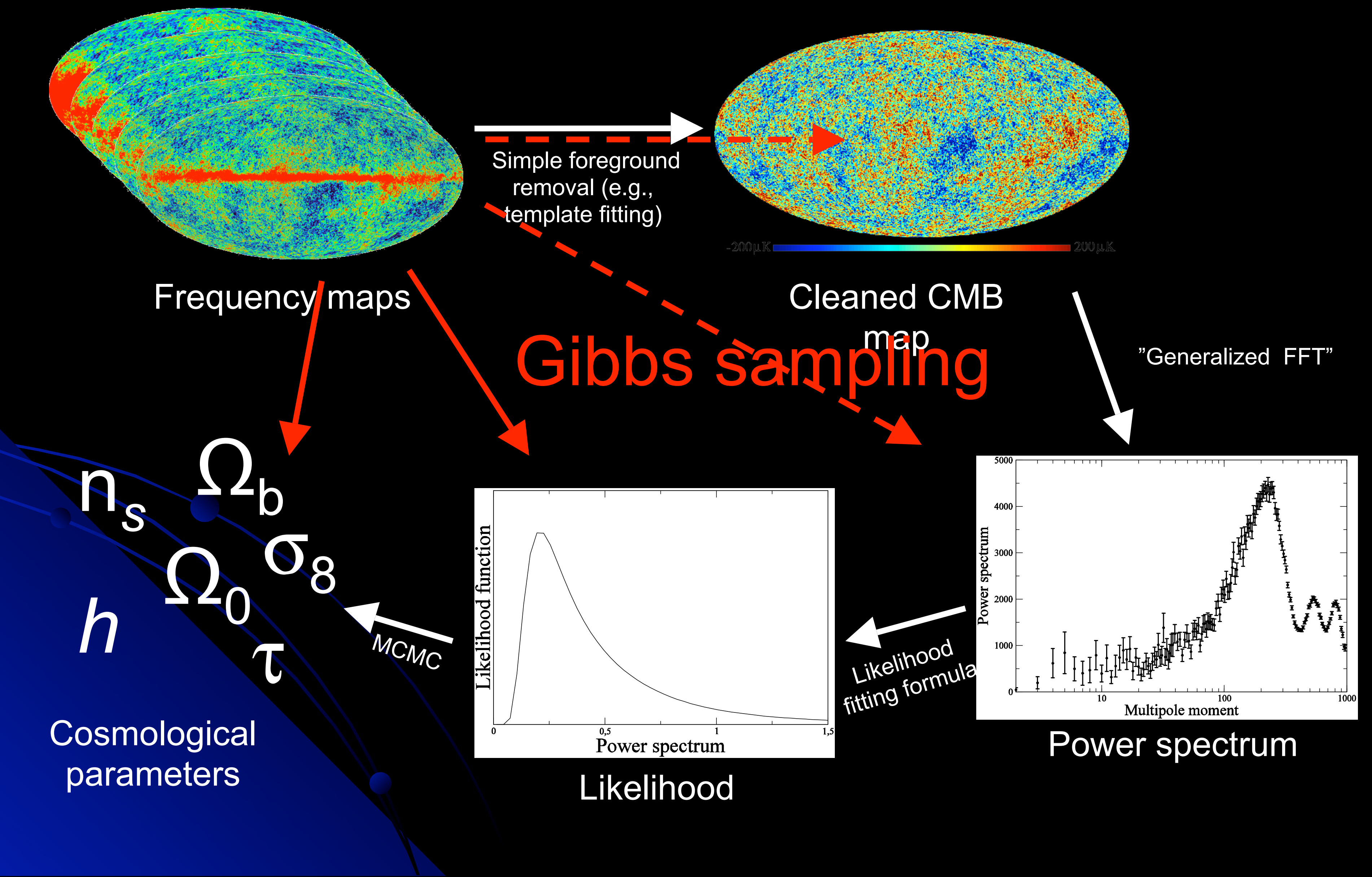

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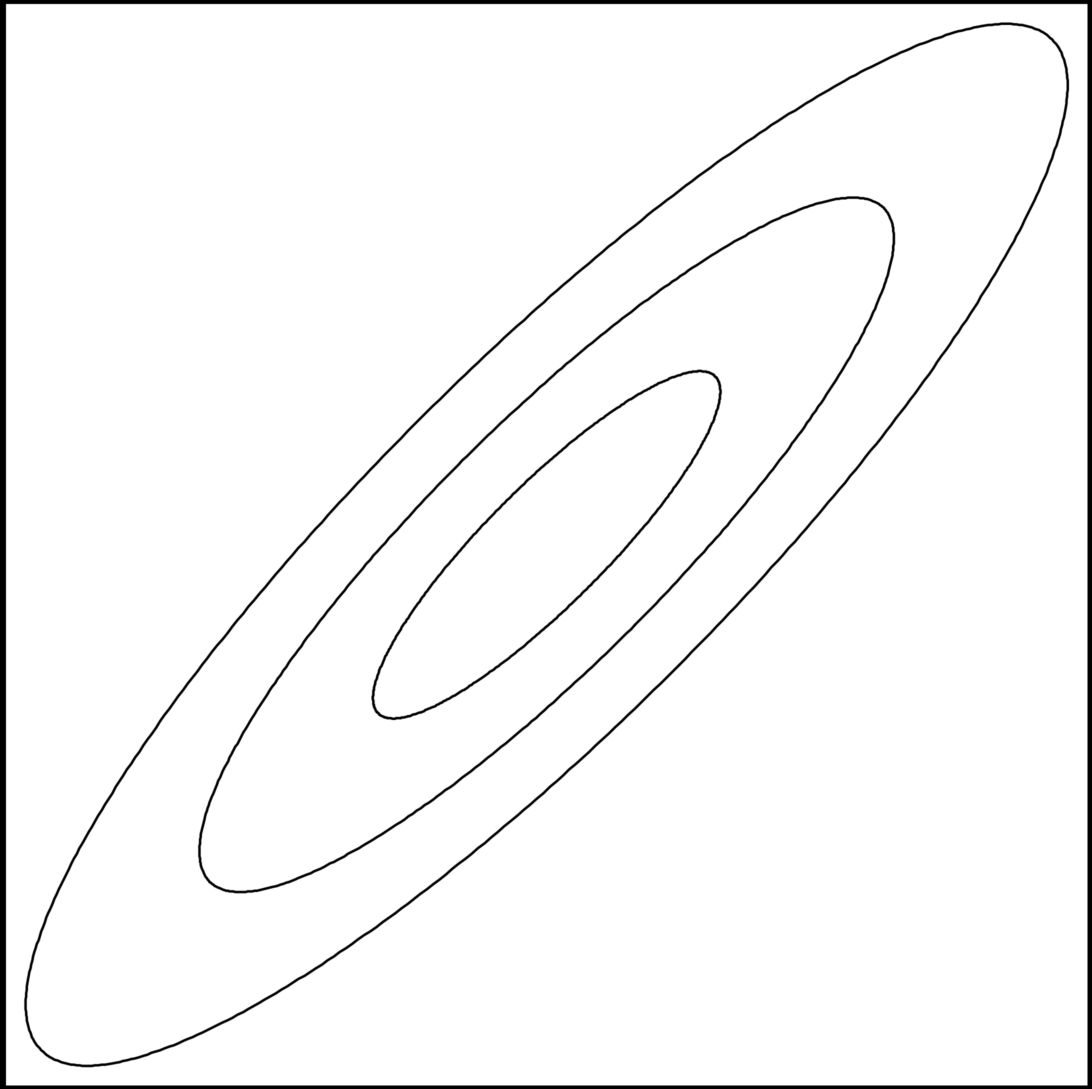
- After some burn-in period, x_i will be drawn from the correct distribution P(x)

Schematic view of MH MCMC



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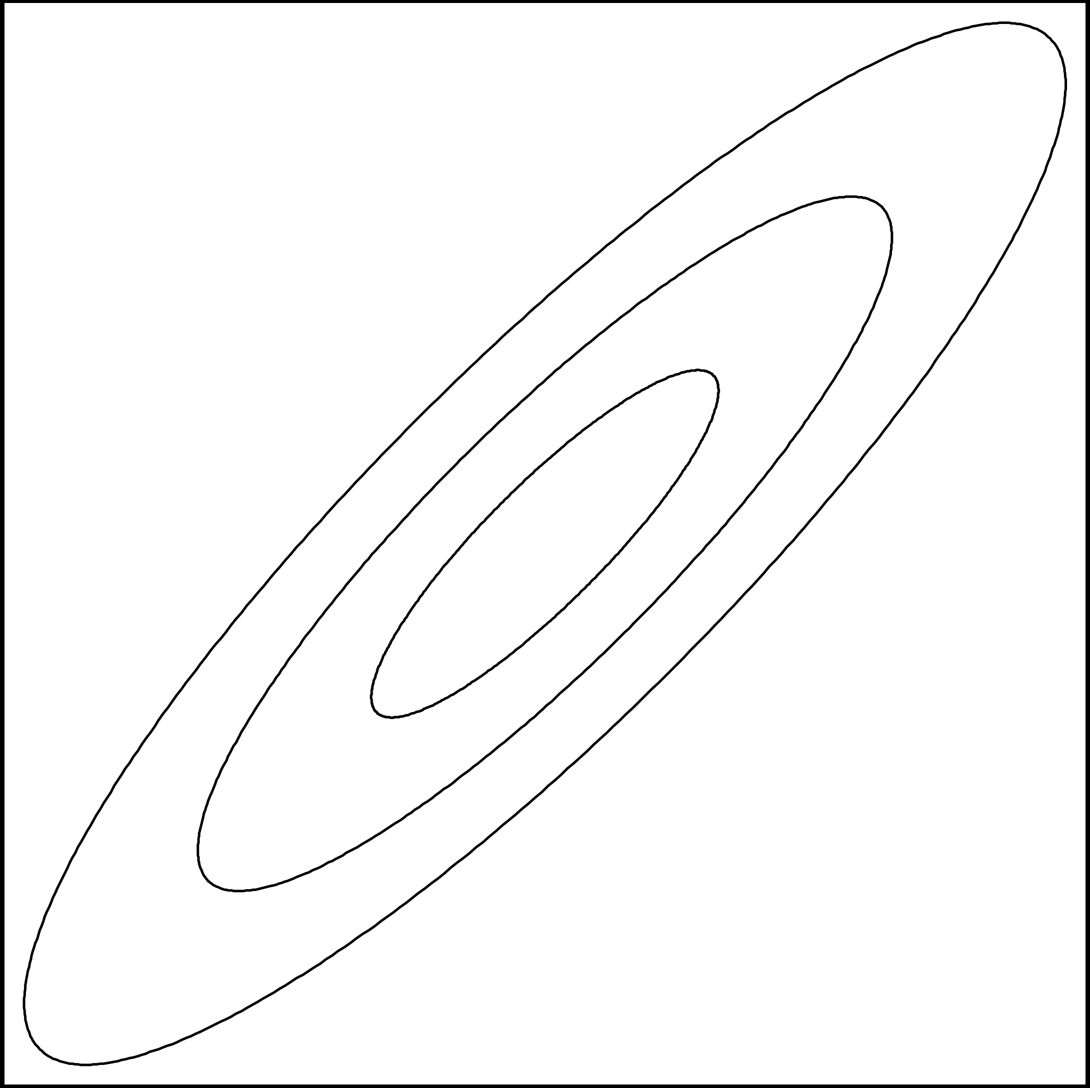

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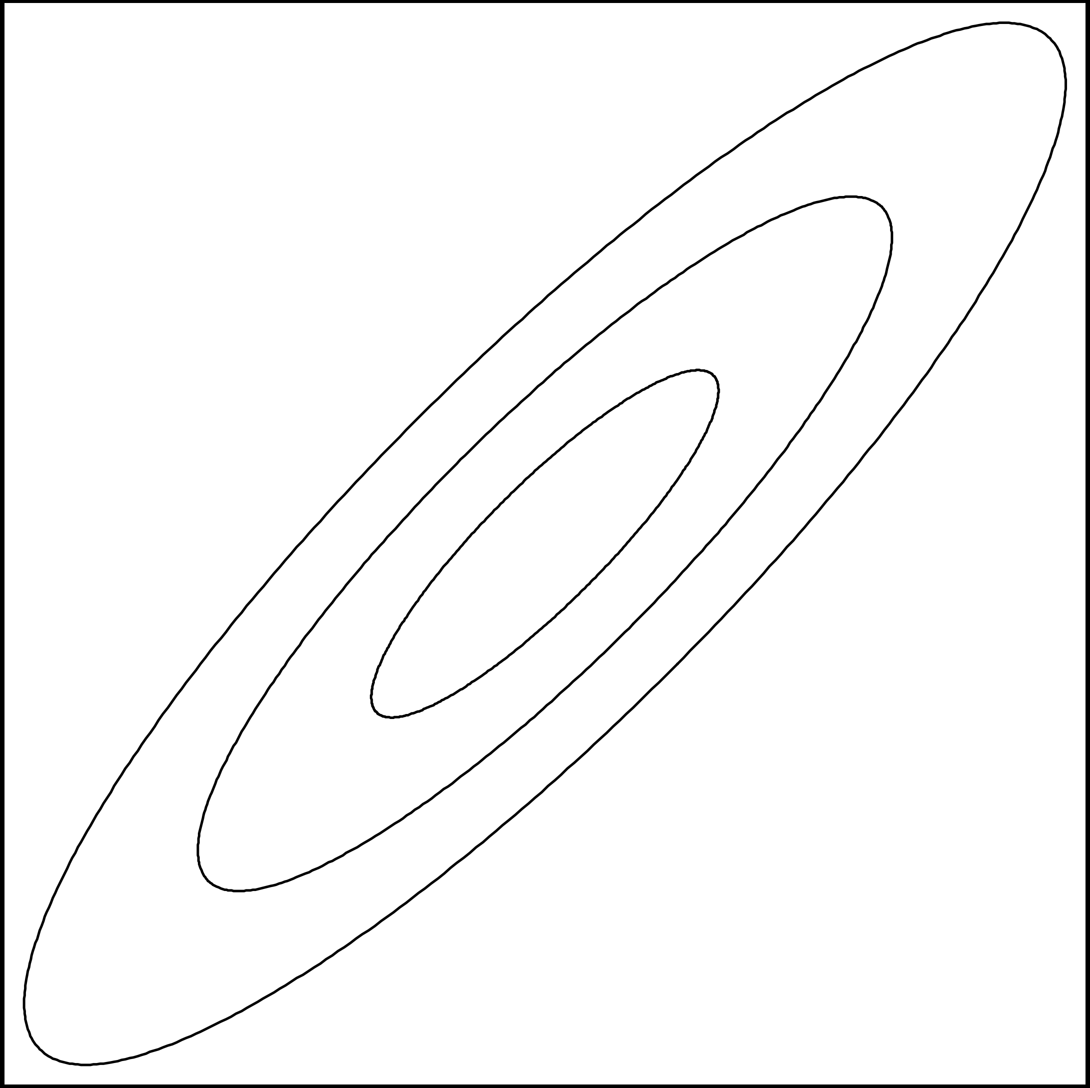

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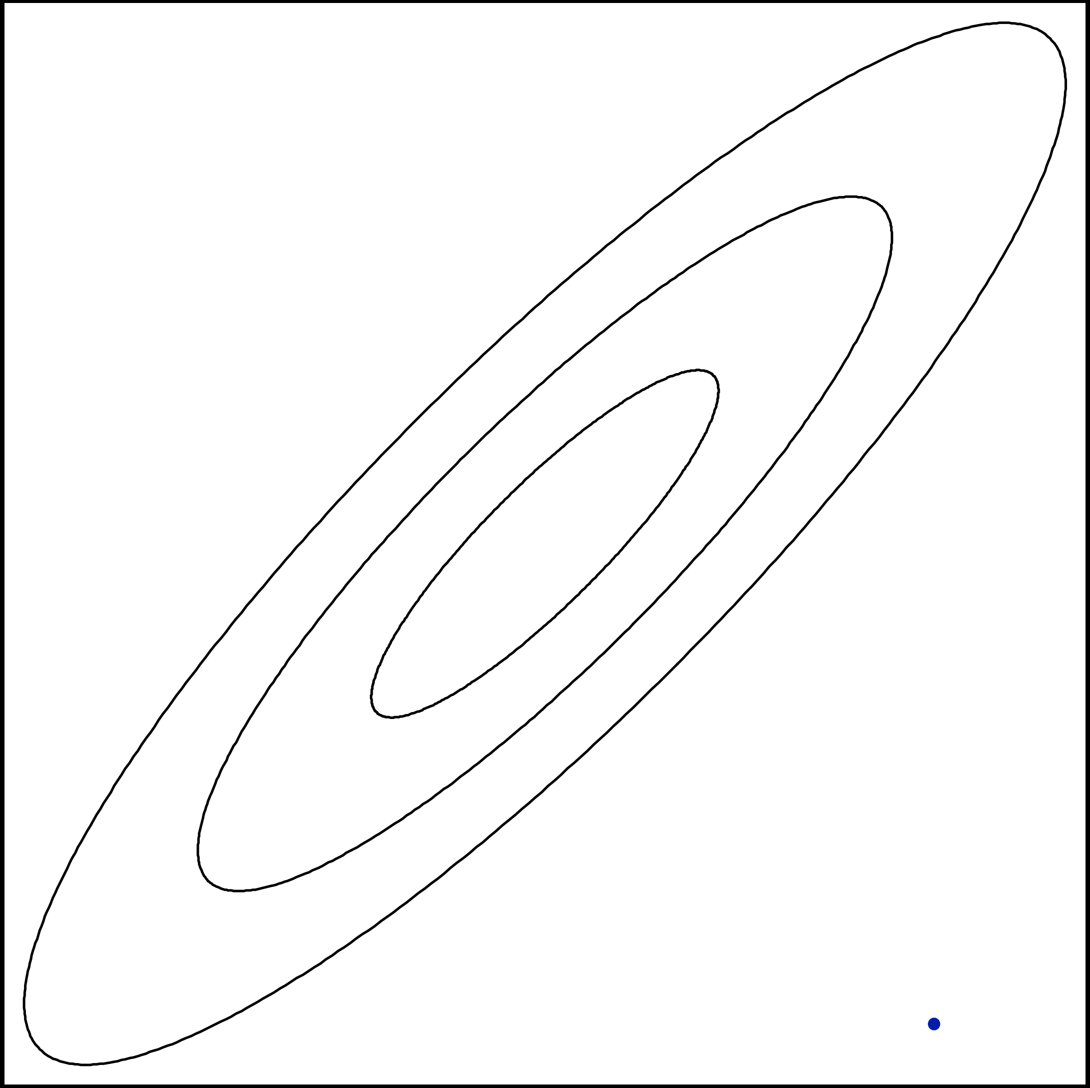

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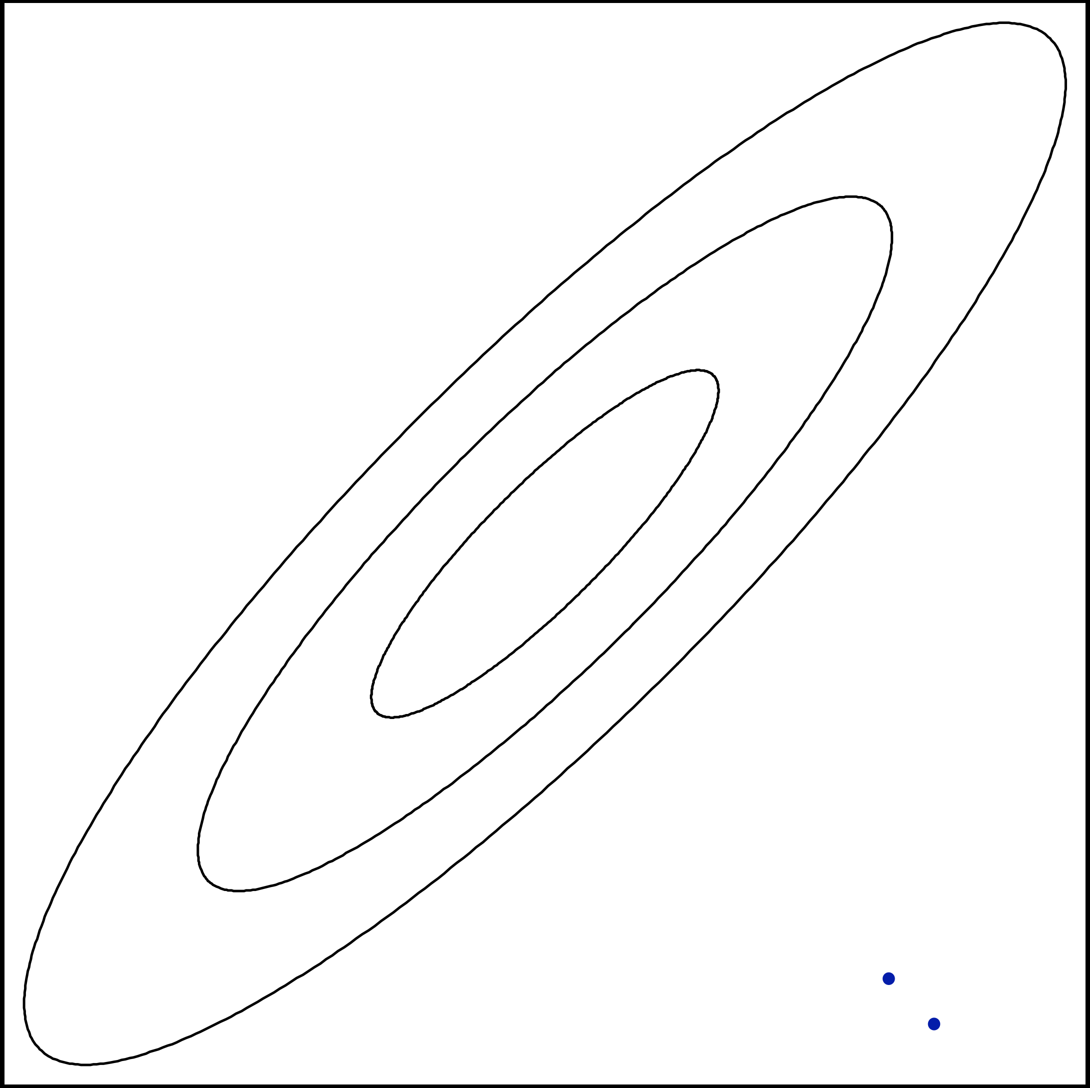

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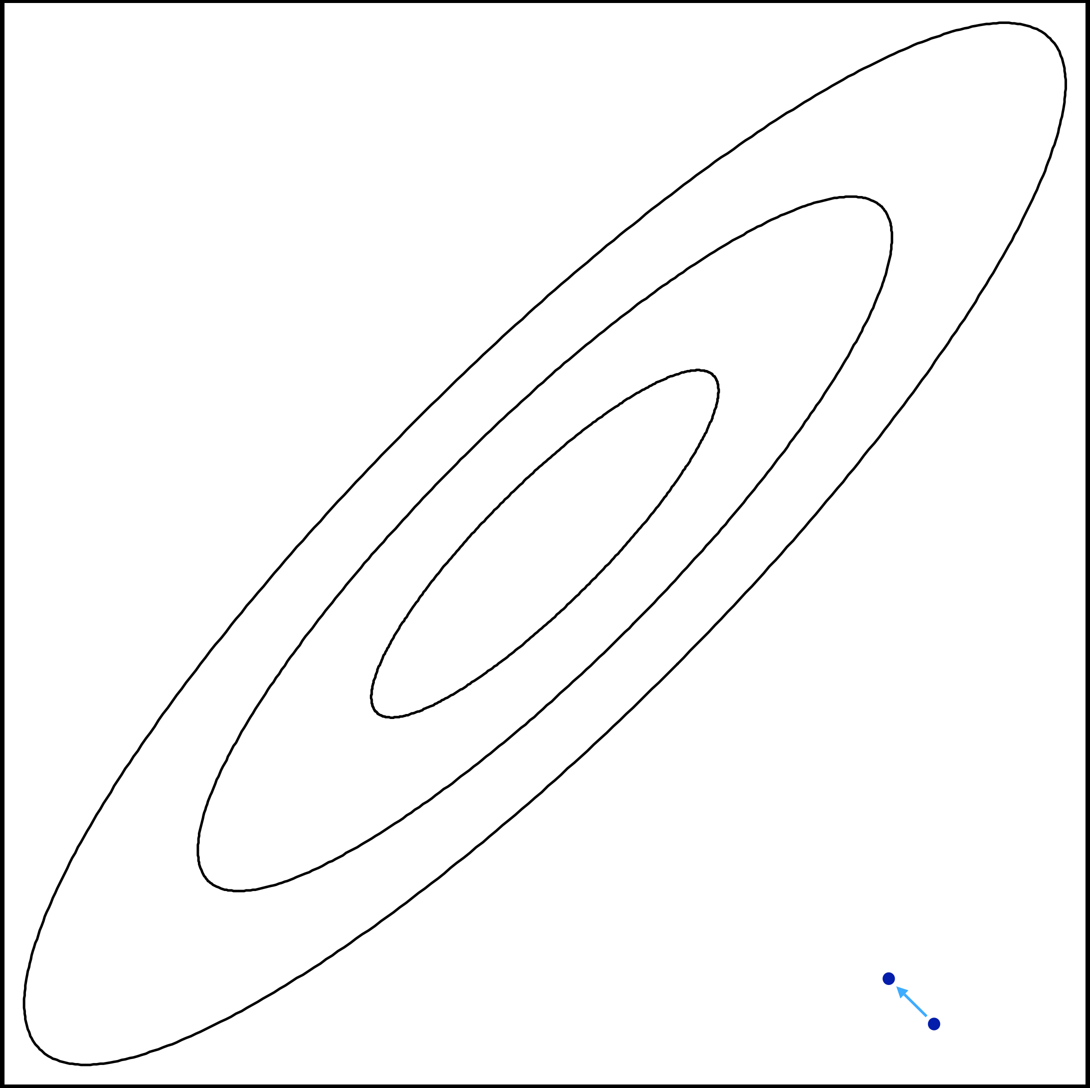

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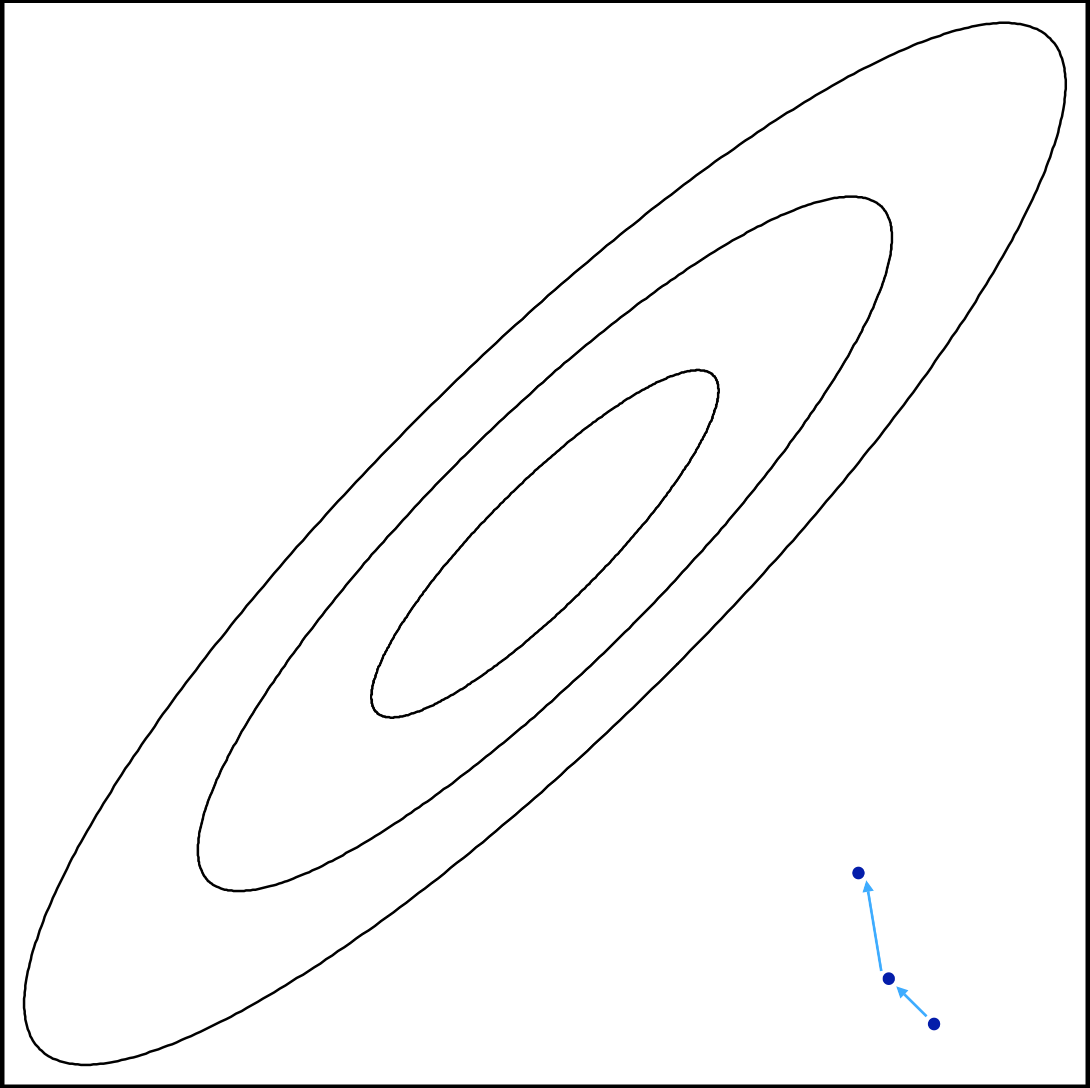

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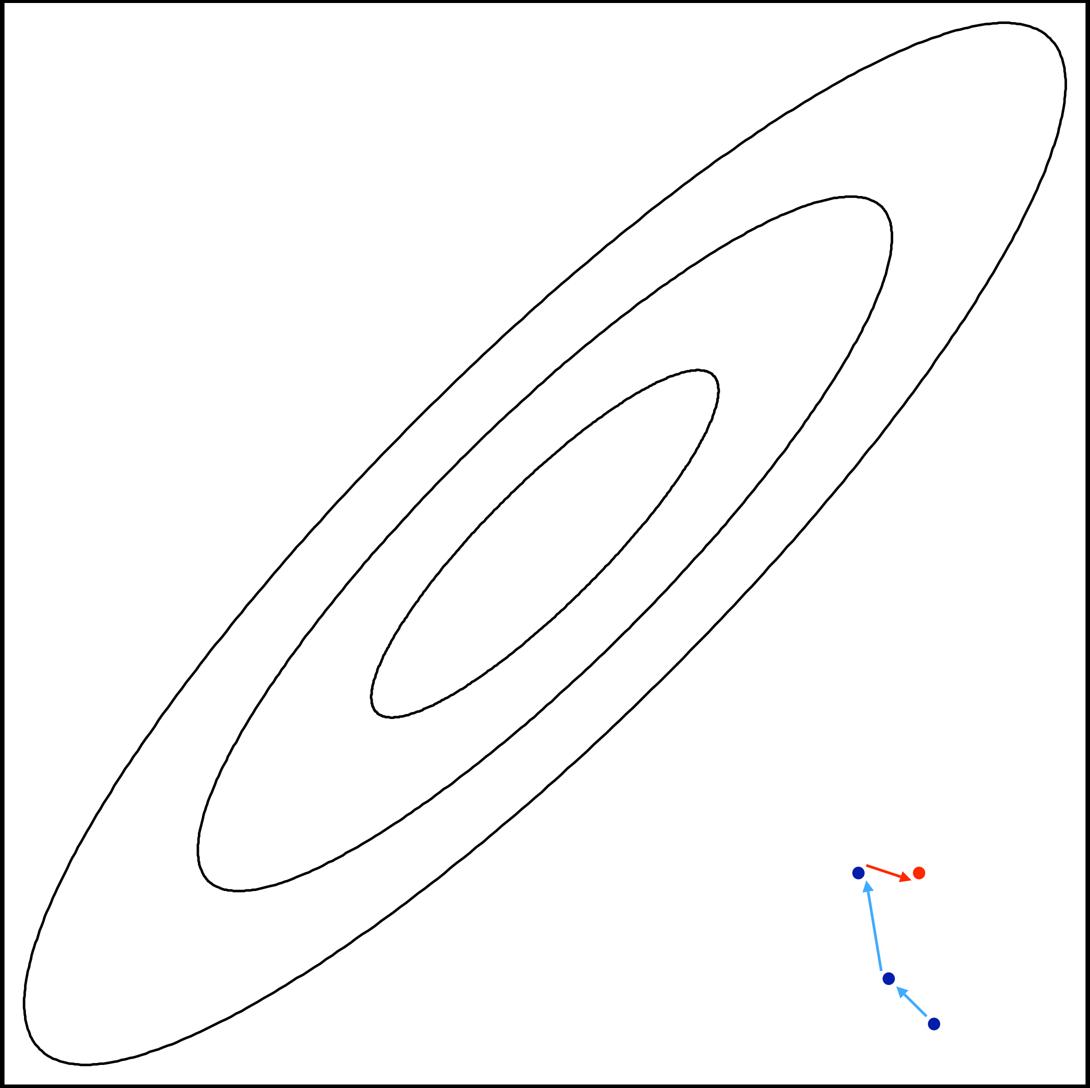

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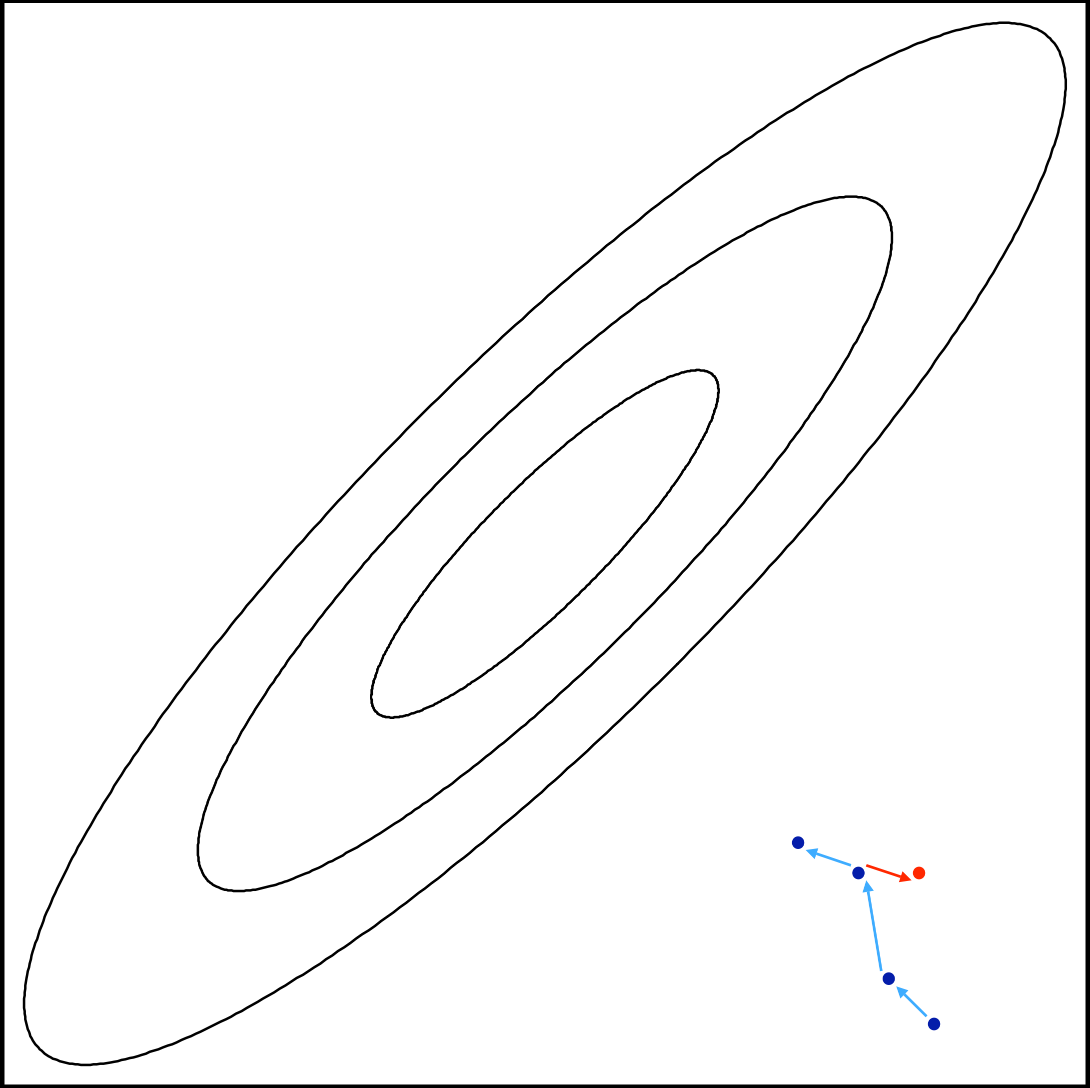

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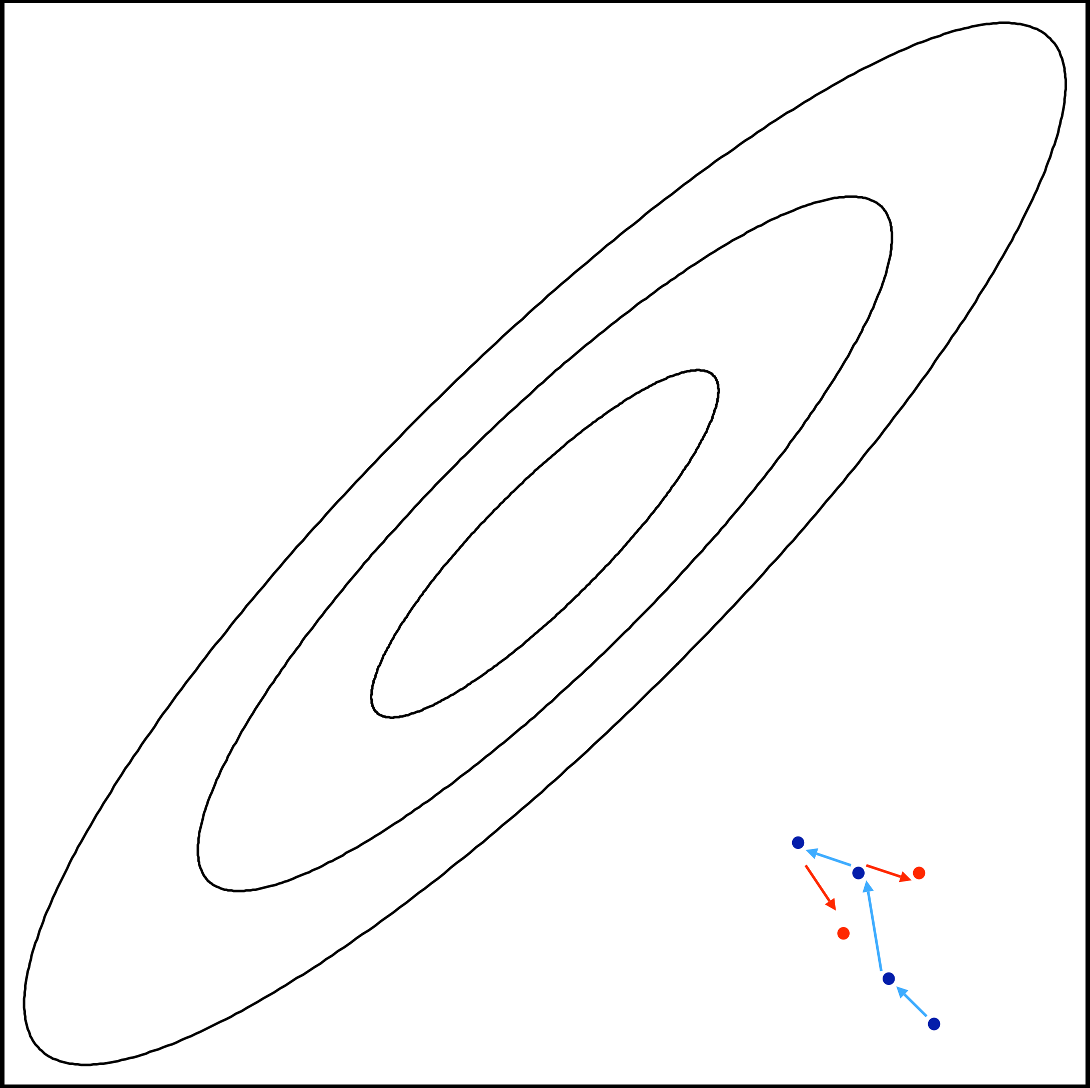

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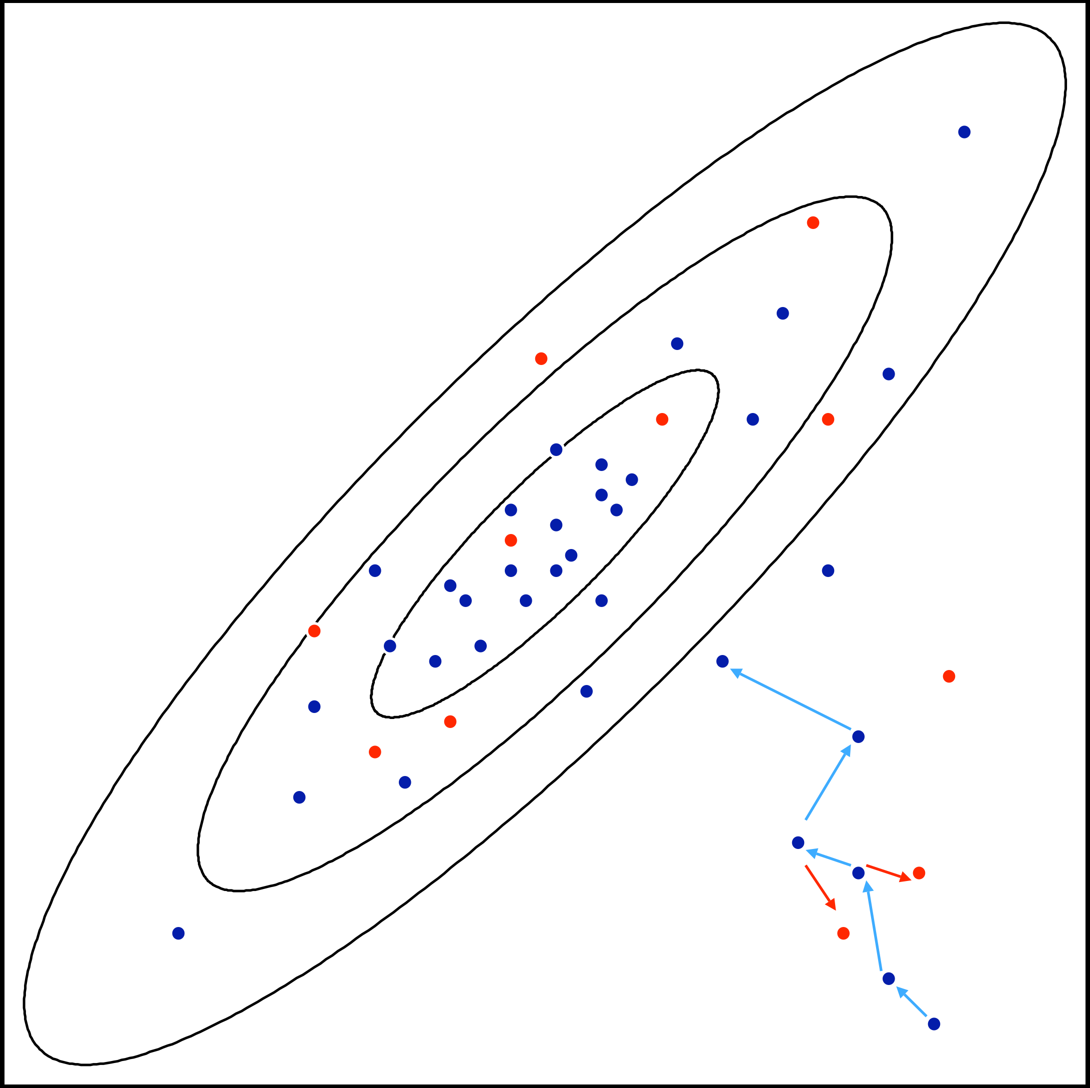

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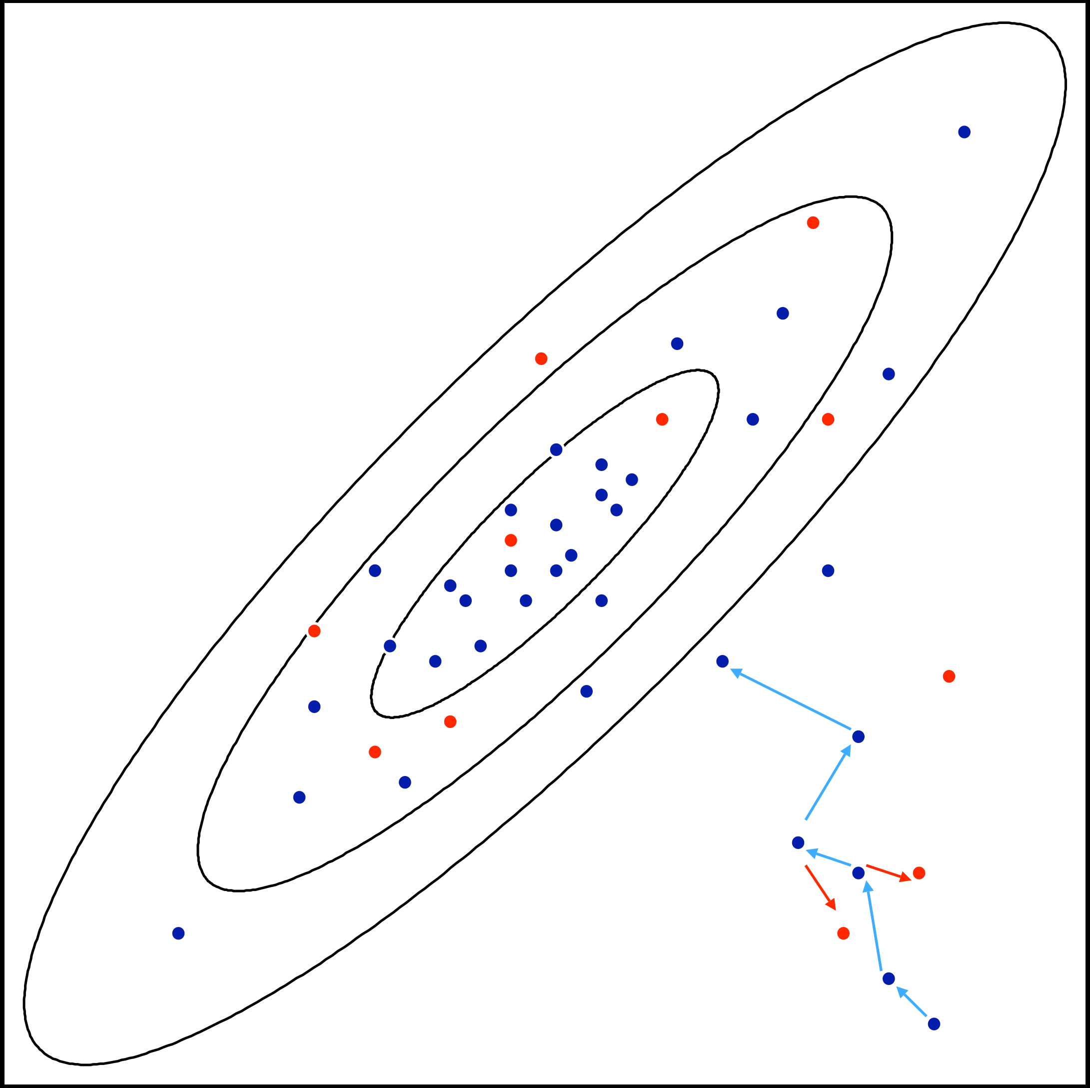

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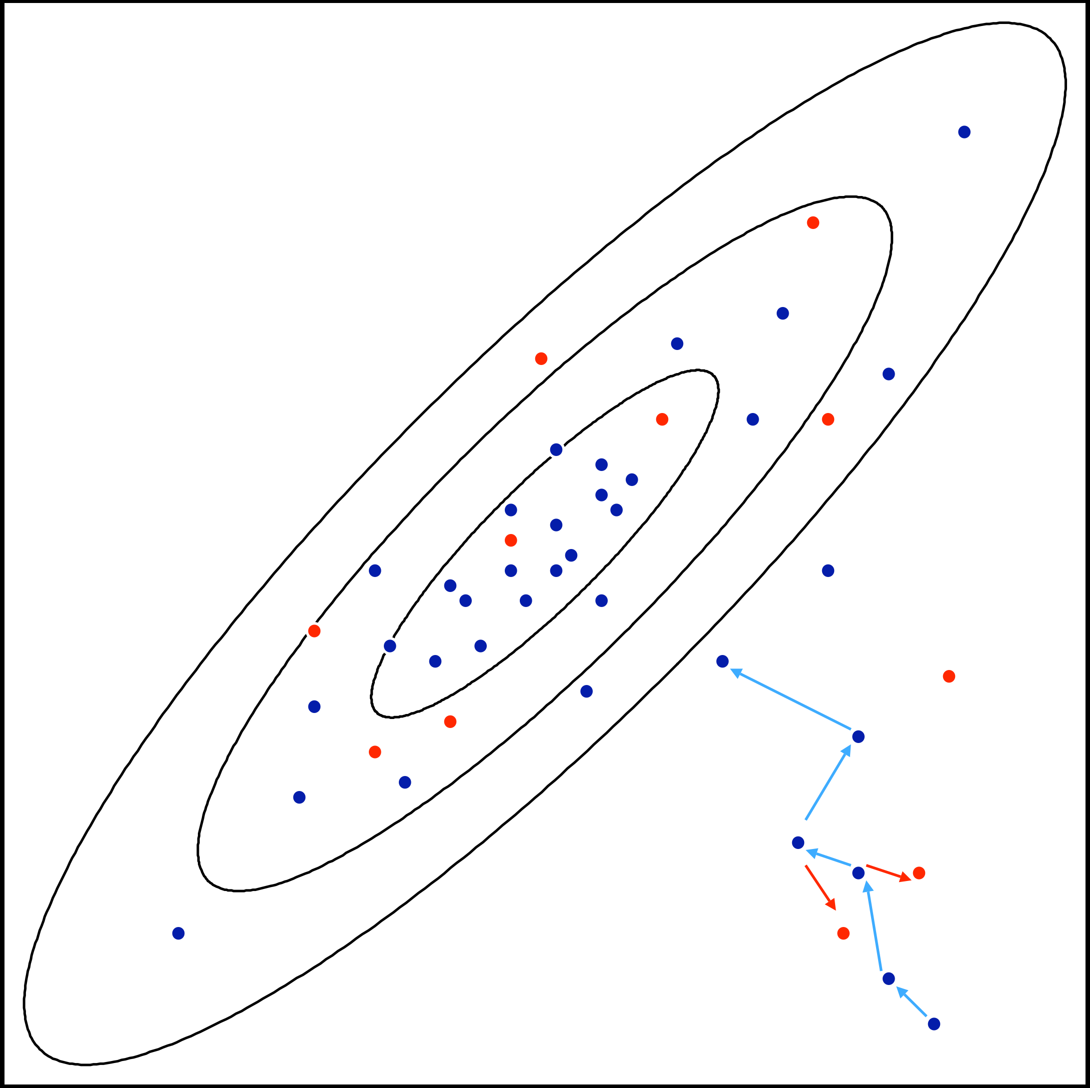
  - Iterate 3 and 4
- Works beautifully for many complex problems, but can be inefficient for many parameters



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- After sampling A given B, we then turn around, and sample B given A

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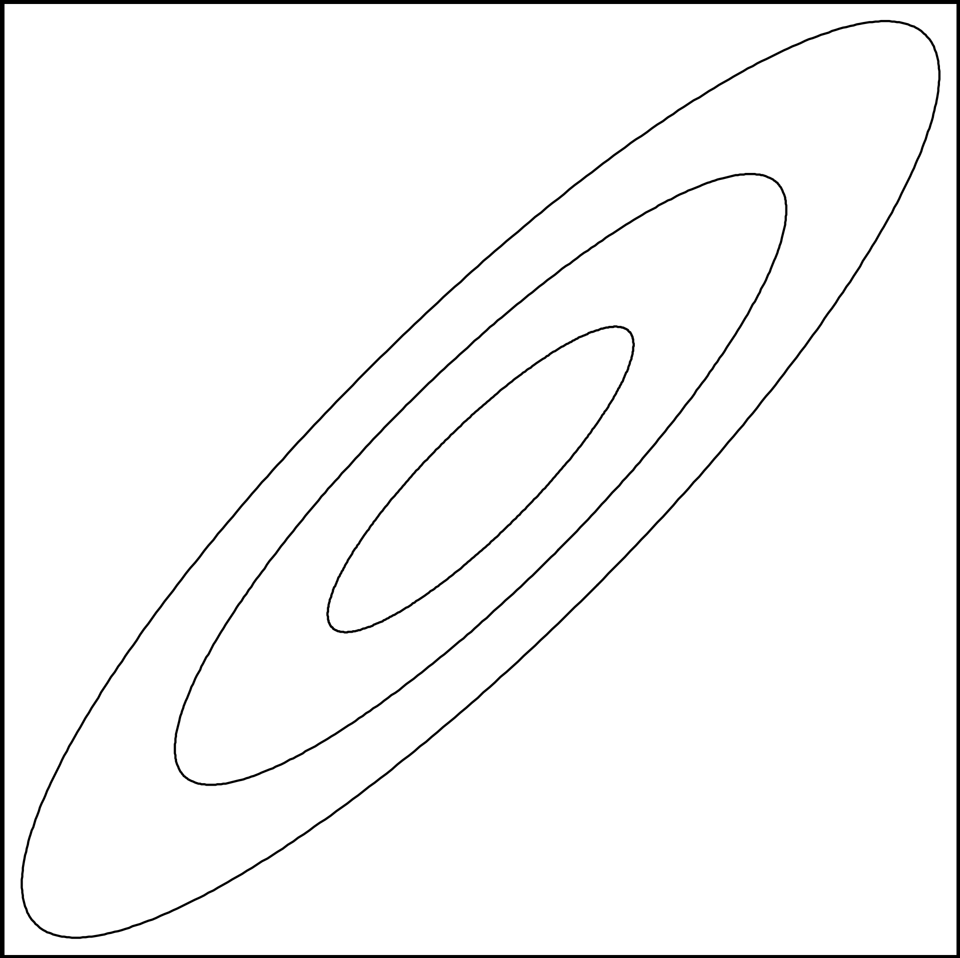

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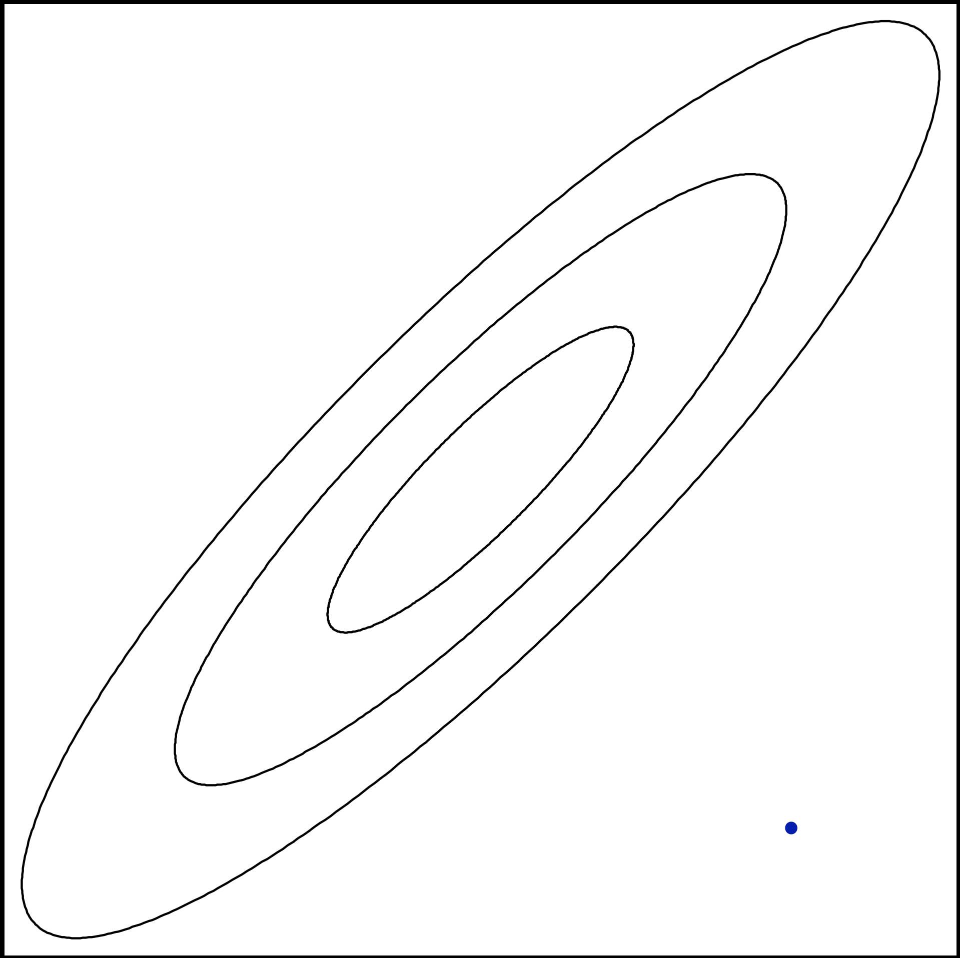

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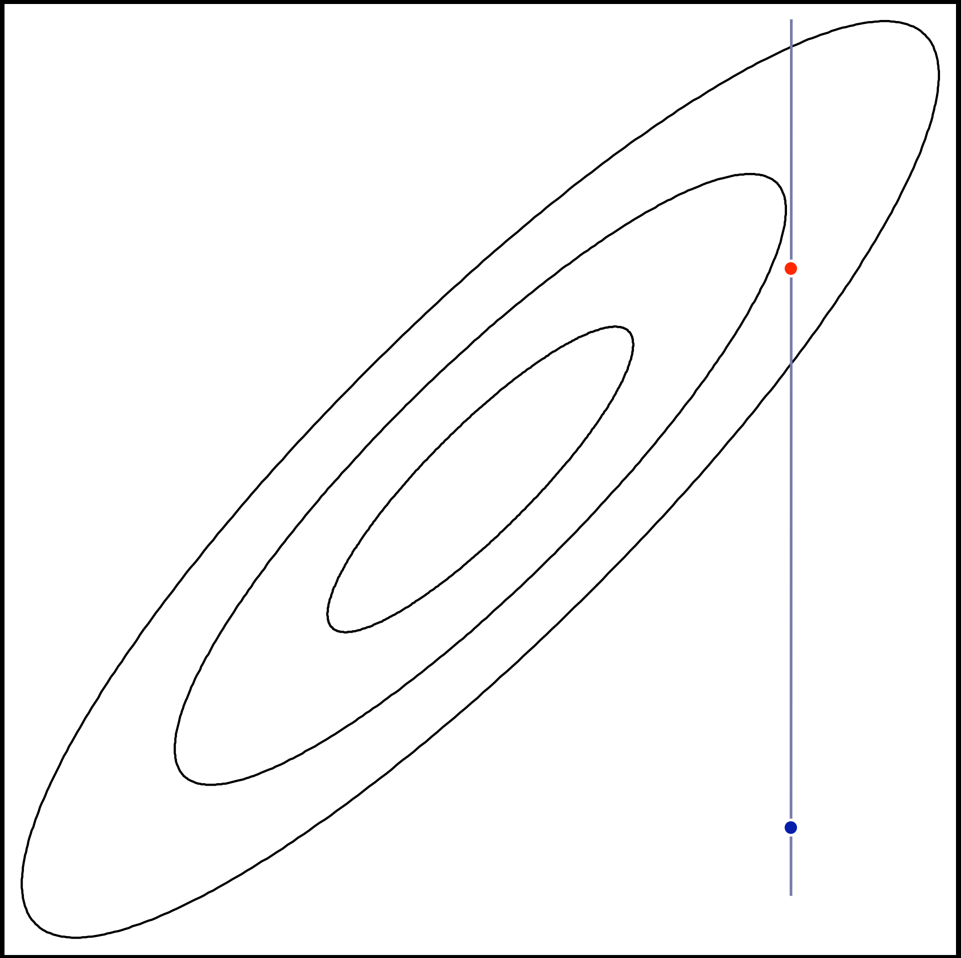

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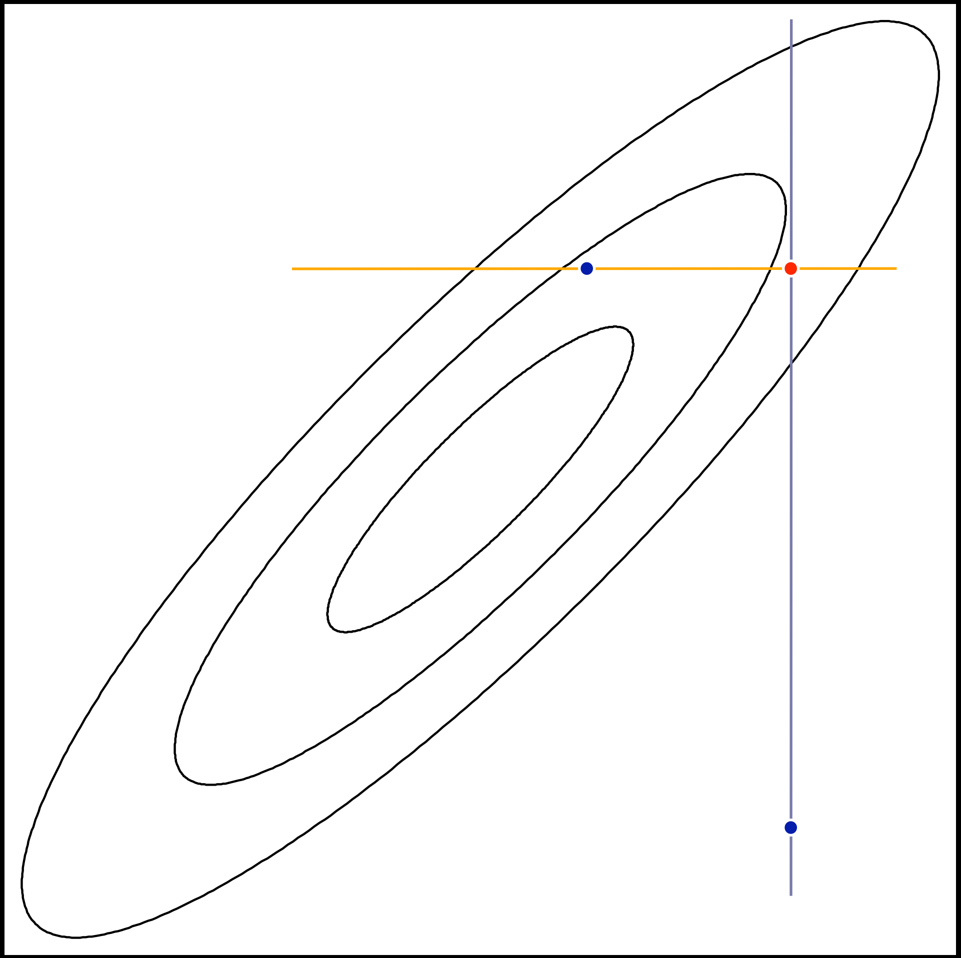

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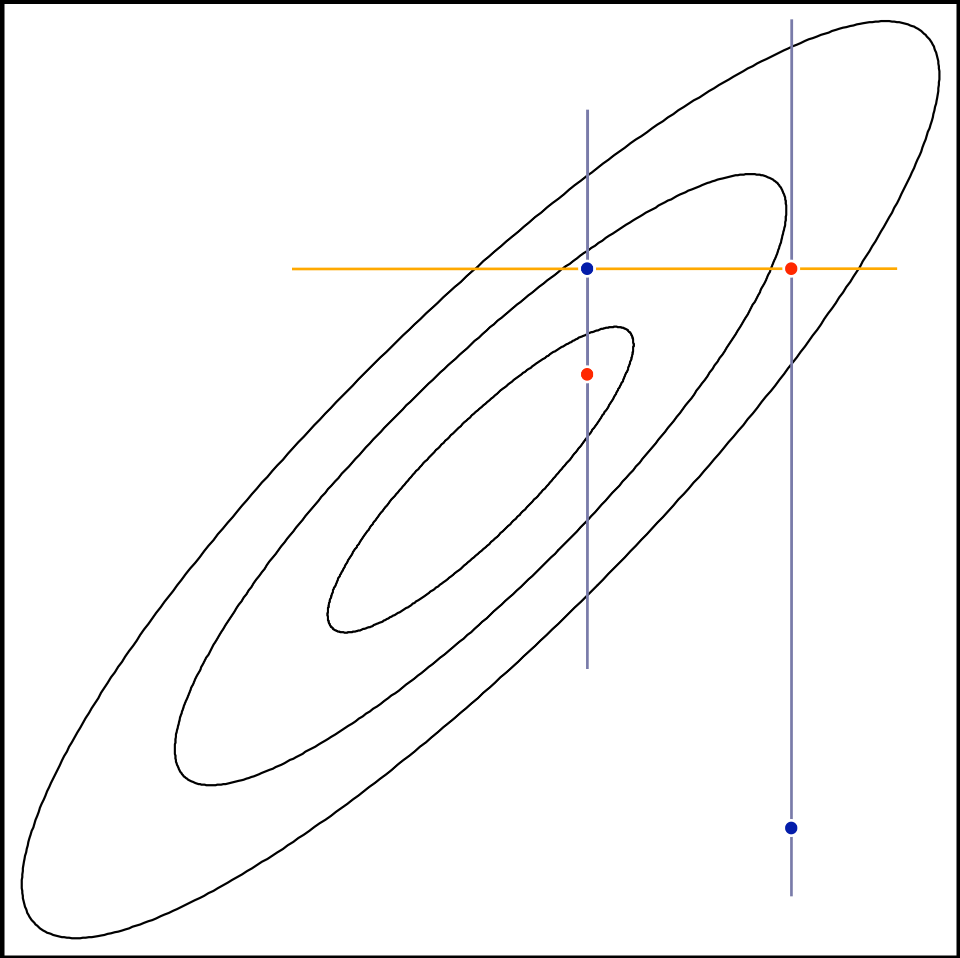

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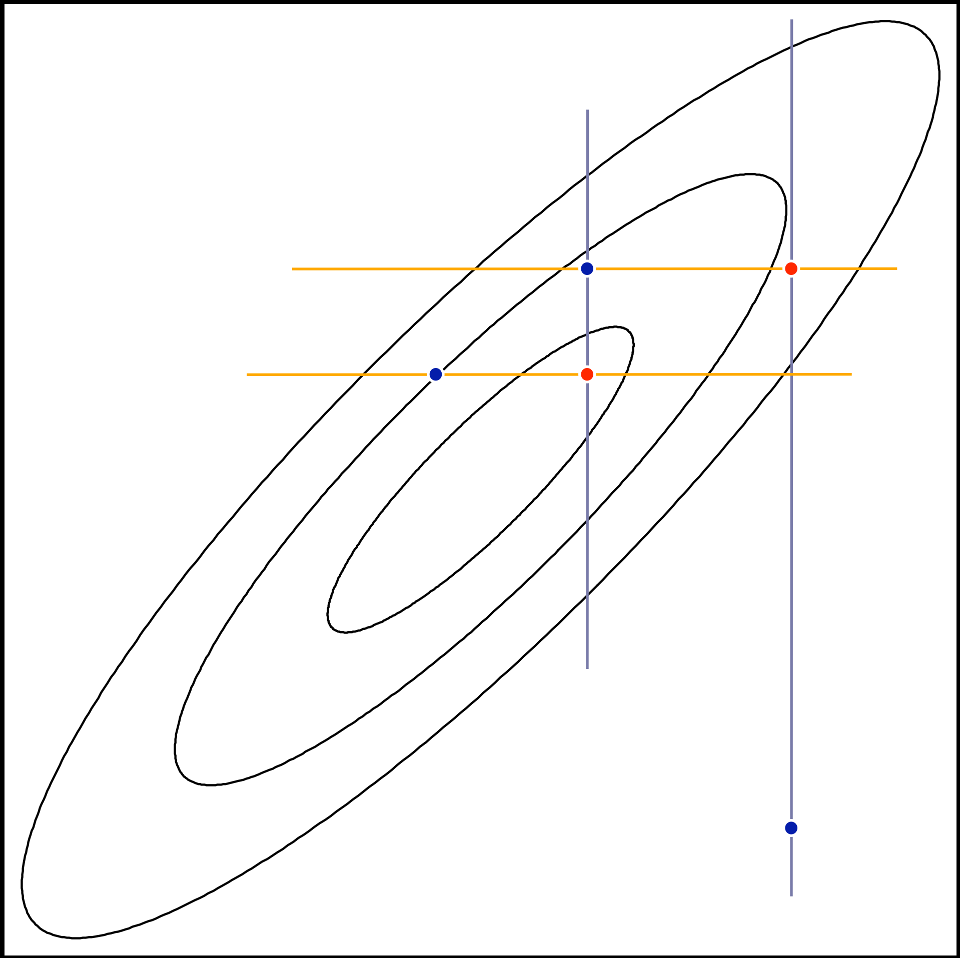

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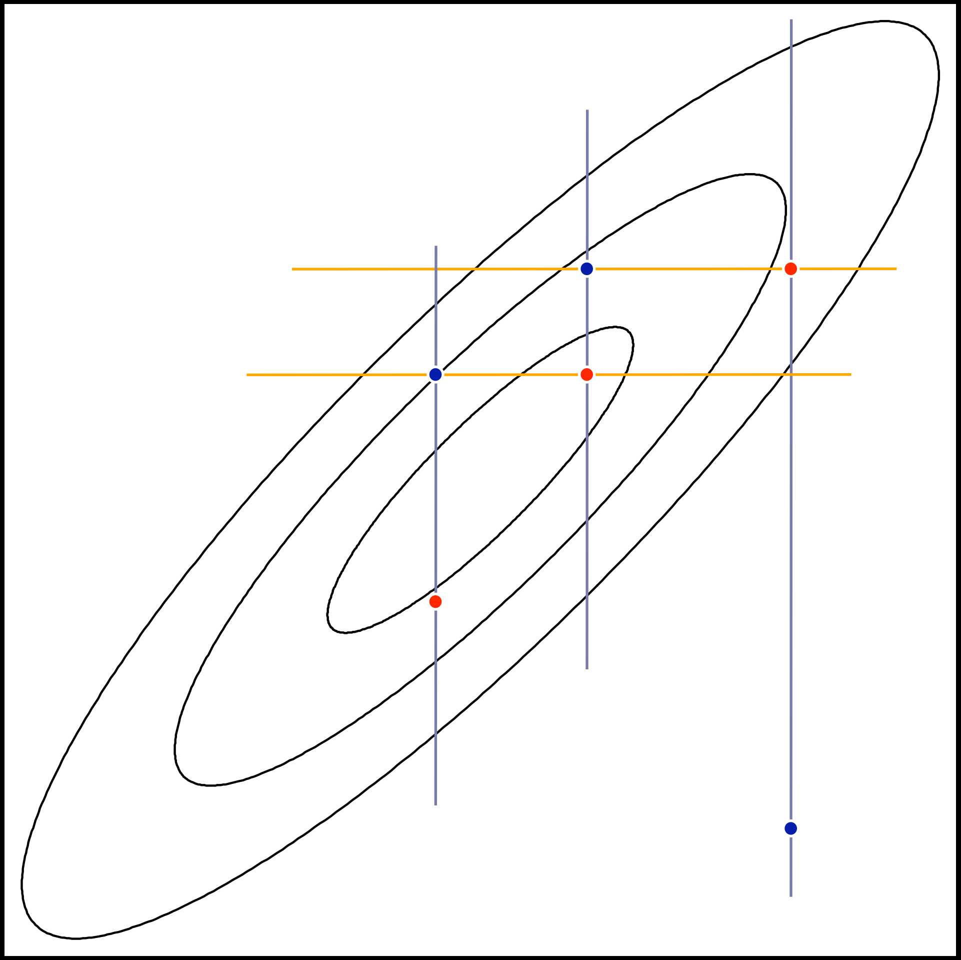

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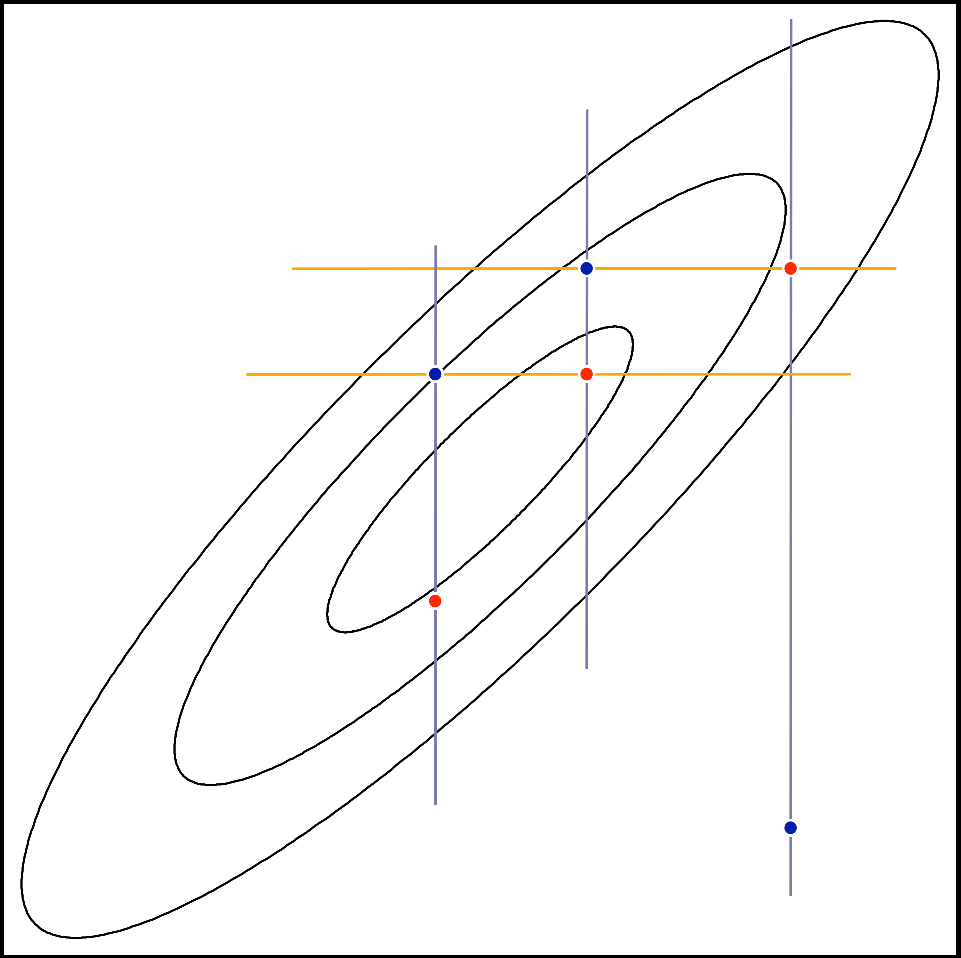

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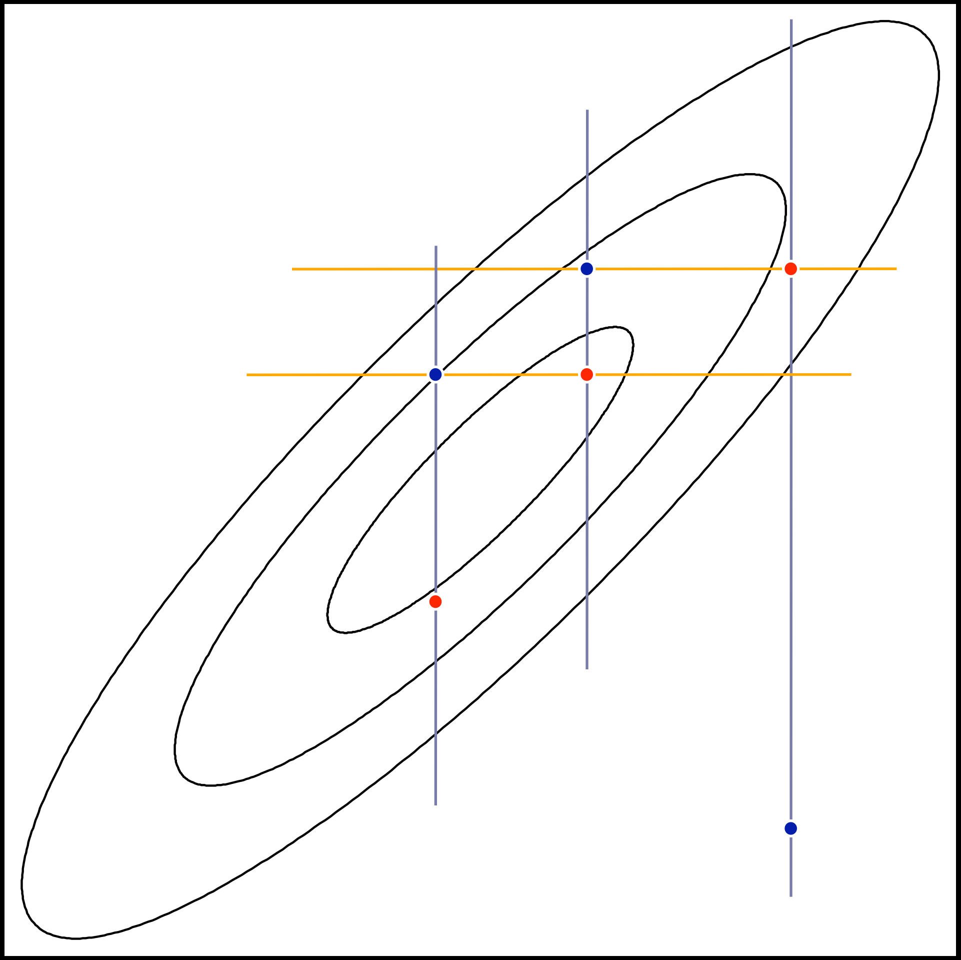

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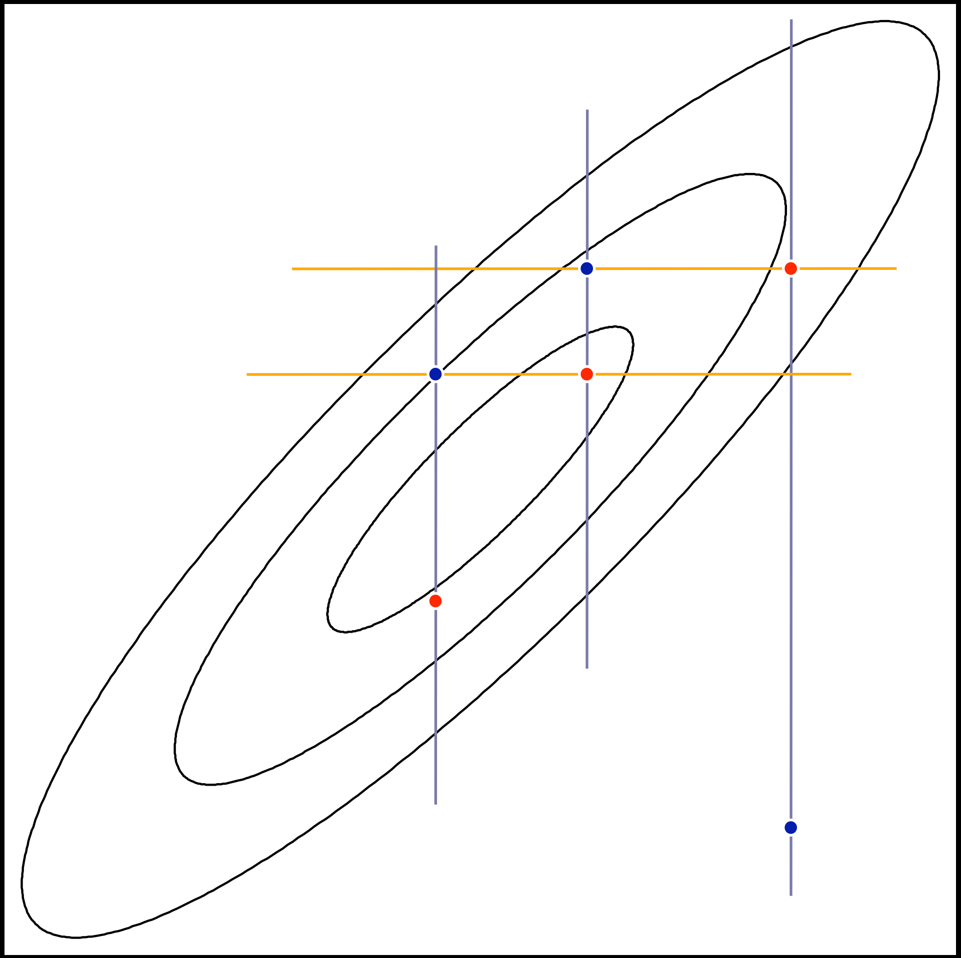

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- Our goal is to extend this framework to take into account all relevant sources of uncertainty, and especially non-cosmological foregrounds

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- **d** = Observed data at frequency υ
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- **f** = Foreground sky signal with parameters θ
- **n** = Instrumental noise
- **A** = Point spread function (beam)

Data model including foregrounds

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- We assume both the CMB and noise to be Gaussian distributed, with a CMB covariance matrices defined in harmonic space by

$$\langle \mathbf{s}_\ell \mathbf{s}_\ell^t \rangle = \begin{pmatrix} C_\ell^{\mathrm{TT}} & C_\ell^{\mathrm{TE}} & C_\ell^{\mathrm{TB}} \\ C_\ell^{\mathrm{TE}} & C_\ell^{\mathrm{EE}} & C_\ell^{\mathrm{EB}} \\ C_\ell^{\mathrm{TB}} & C_\ell^{\mathrm{EB}} & C_\ell^{\mathrm{BB}} \end{pmatrix} \delta_{\ell\ell}$$

The CMB-foreground Gibbs sampler

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Gaussian; sampled as before

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- Explicitly, while the CMB sampler is virtually unchanged, the foreground component is sampled based on the χ^2

Example: Simulated Planck data

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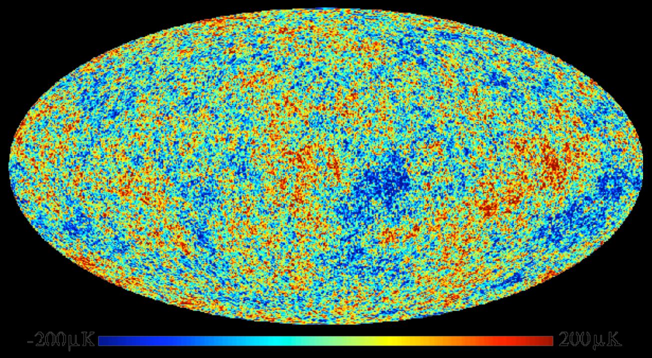
- Sky signal includes all ”known” components (”Planck sky model”):



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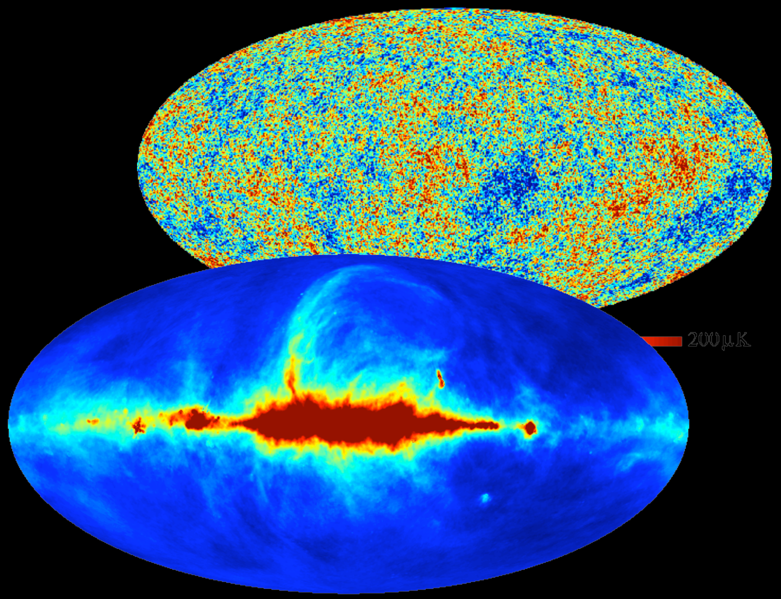

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- Sky signal includes all ”known” components (”Planck sky model”):
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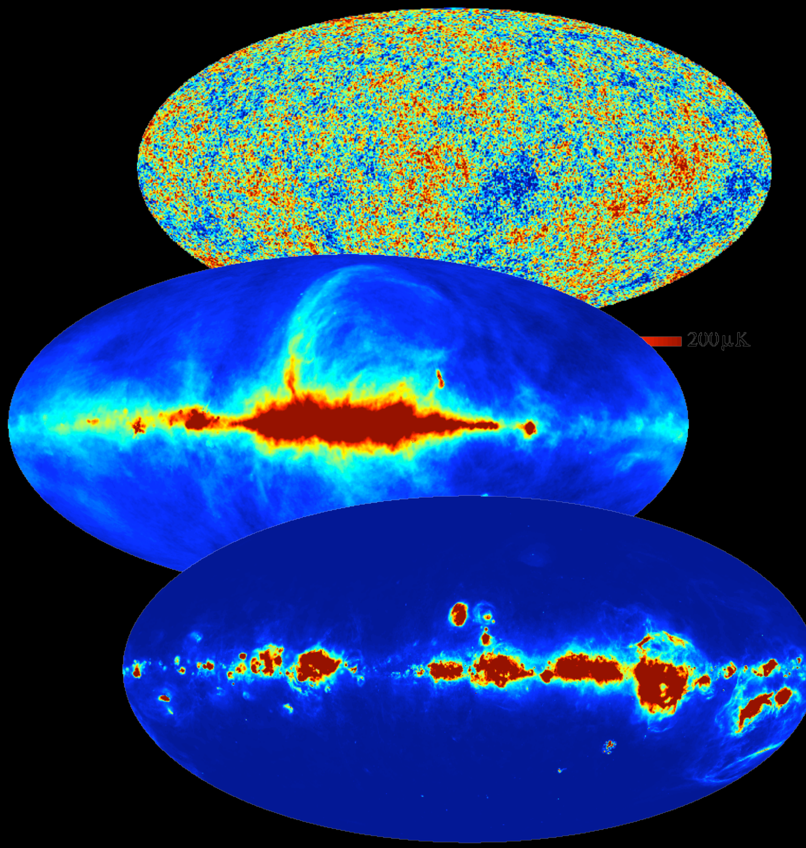

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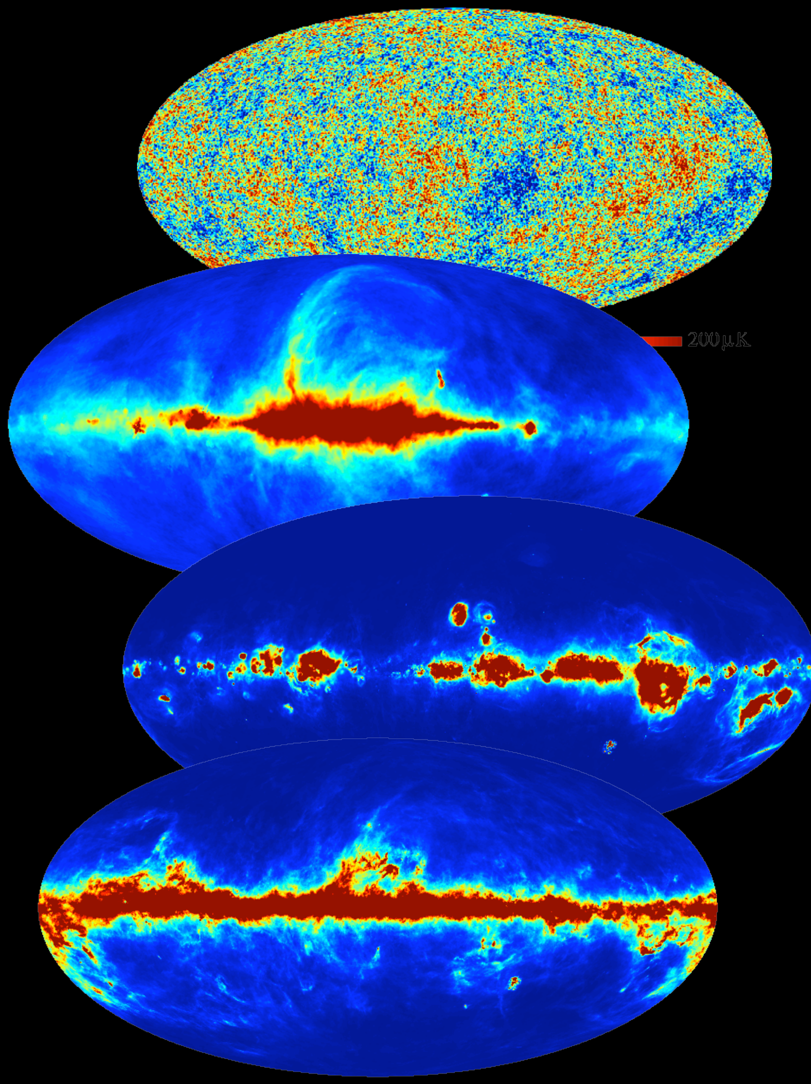

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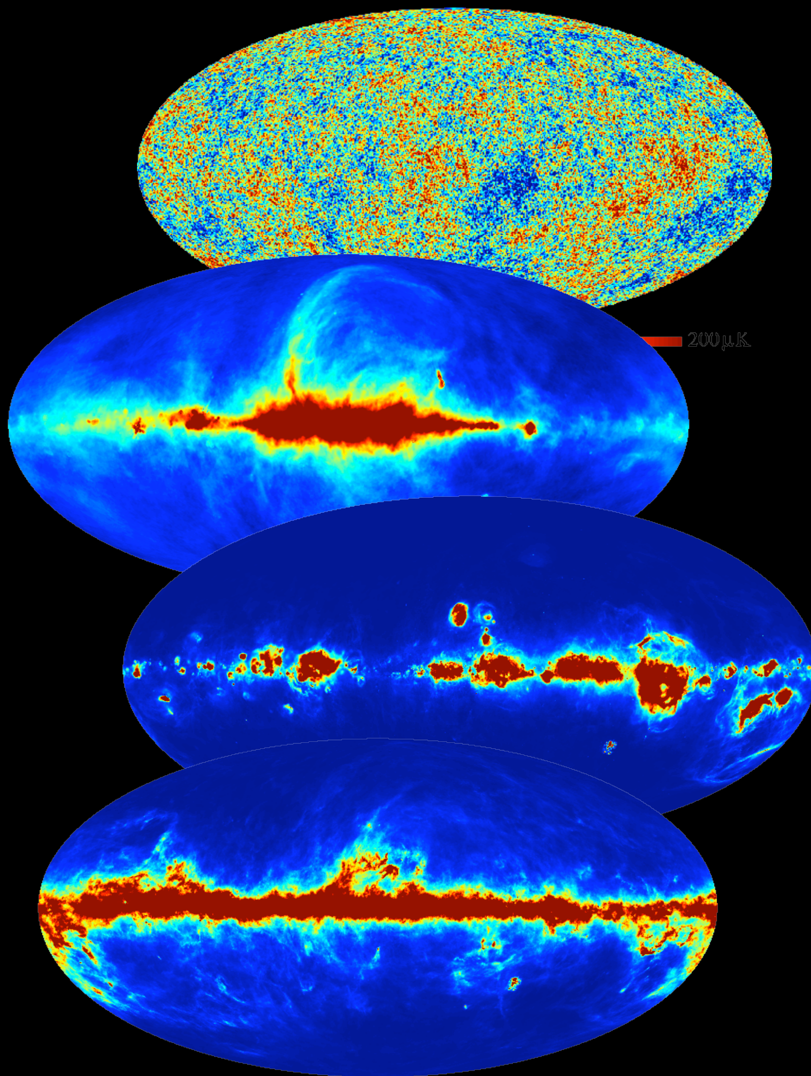

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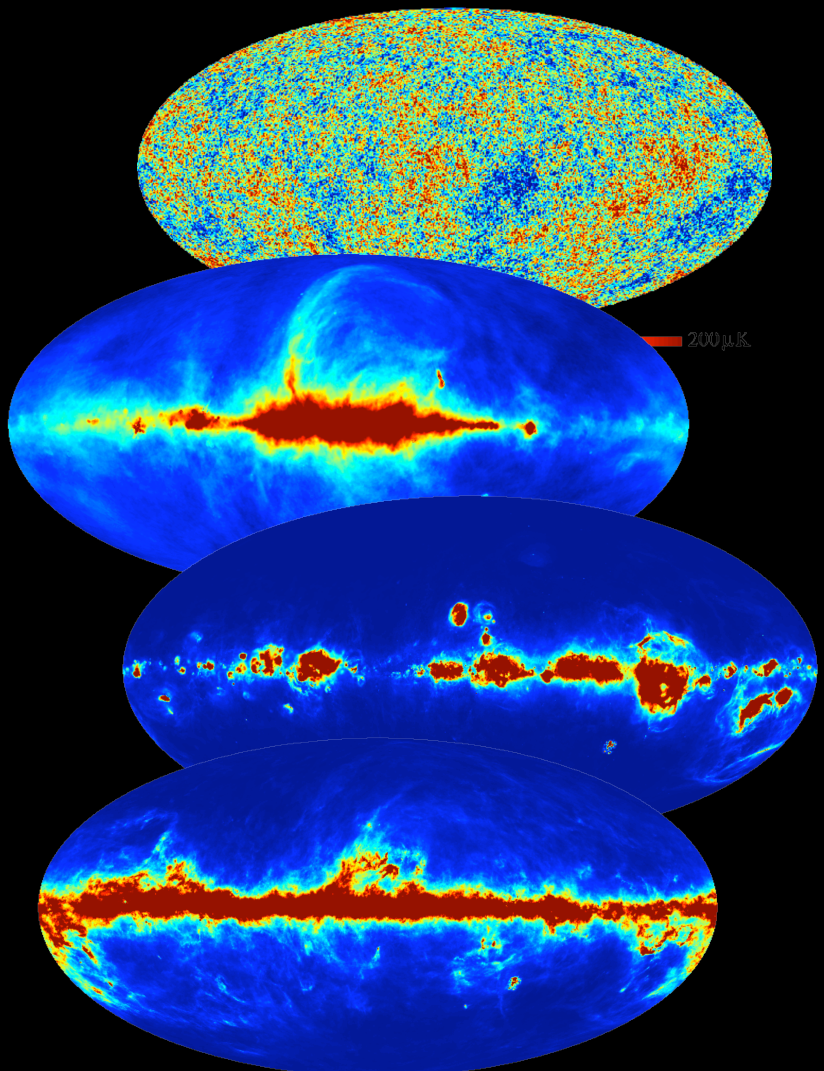

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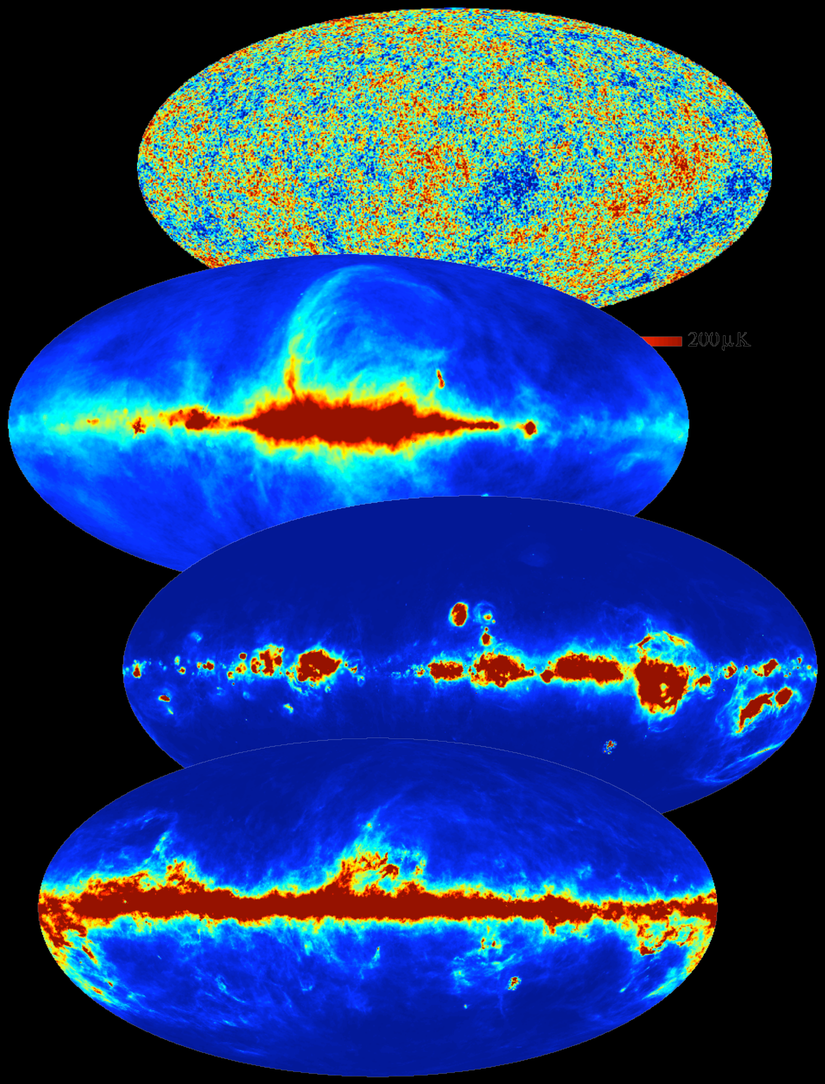

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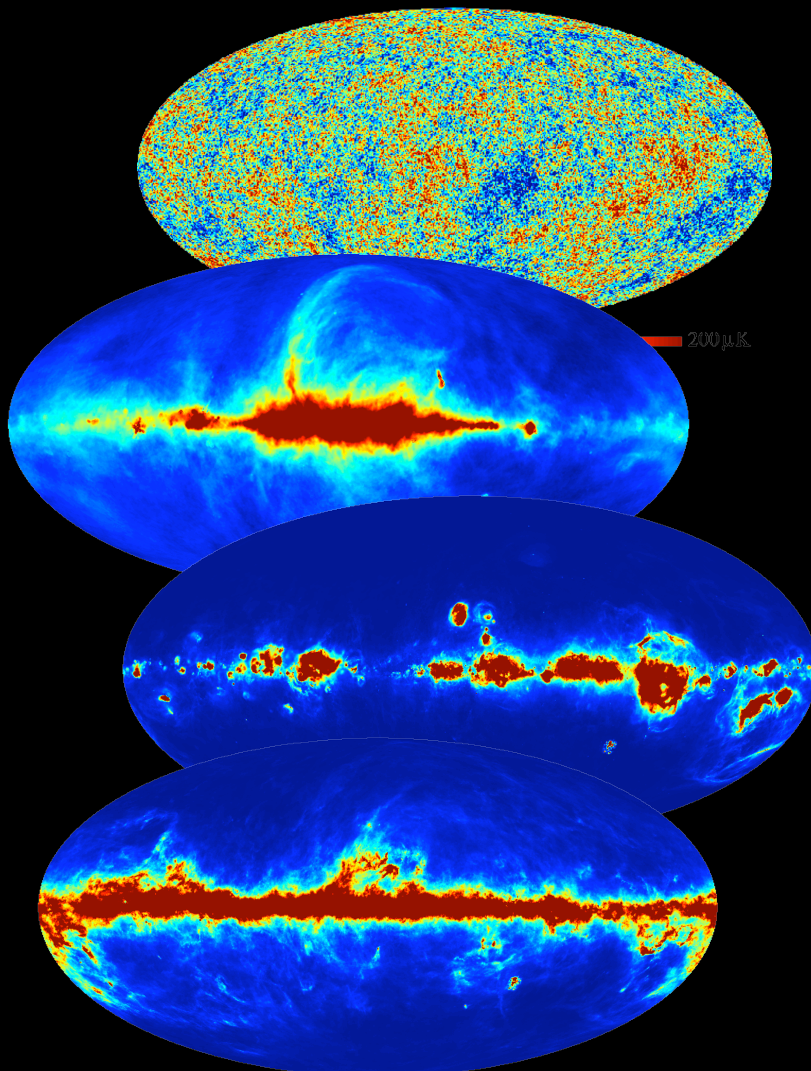
  - The Sunyaev-Zeldovich effect
  - Three populations of point sources



Simulated sky maps

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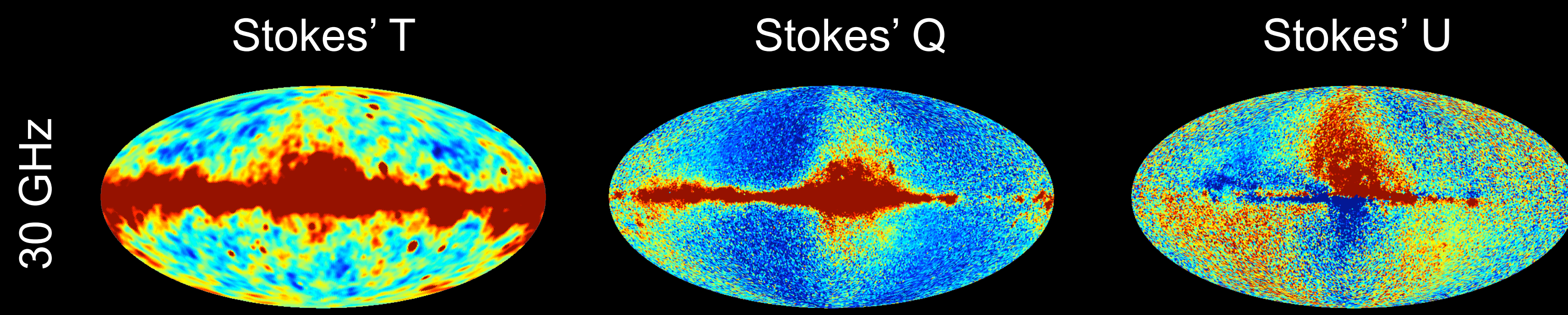
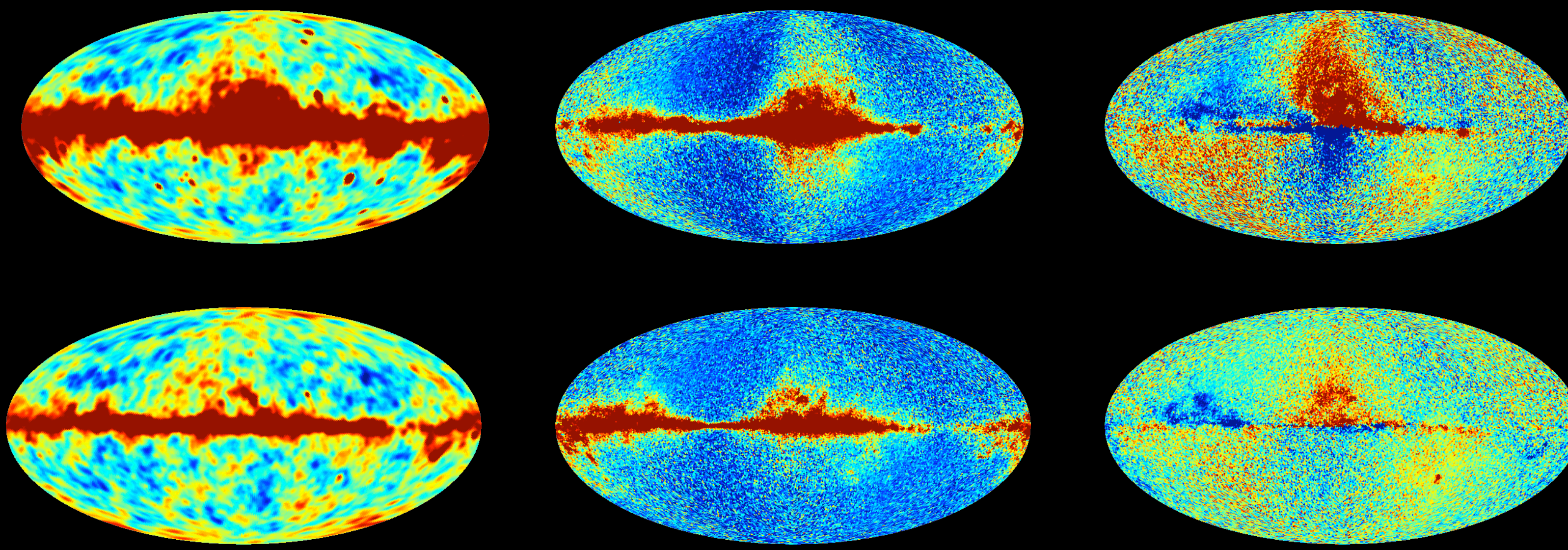


Simulated sky maps

Stokes' T Stokes' Q Stokes' U

30 GHz

100 GHz



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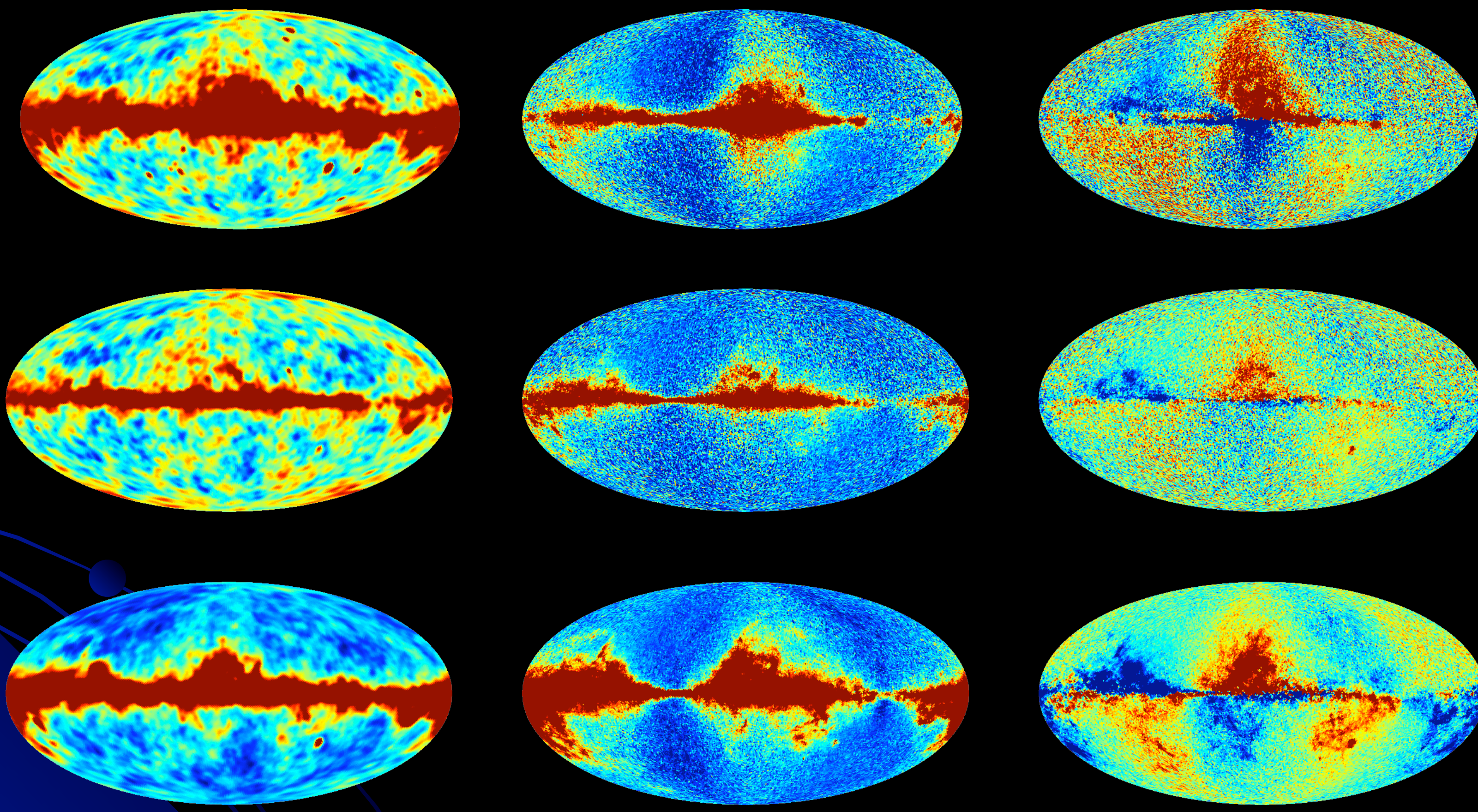

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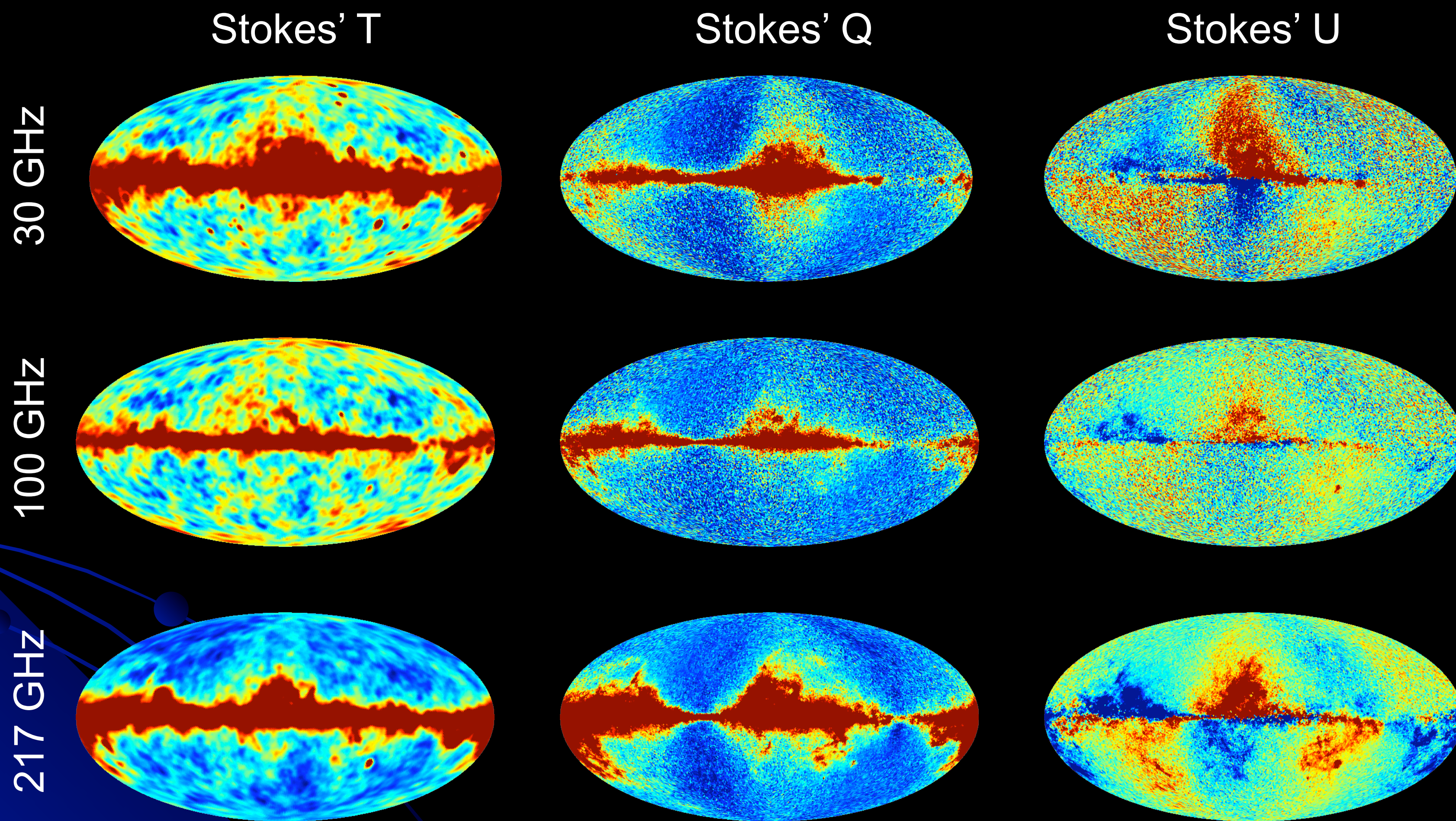
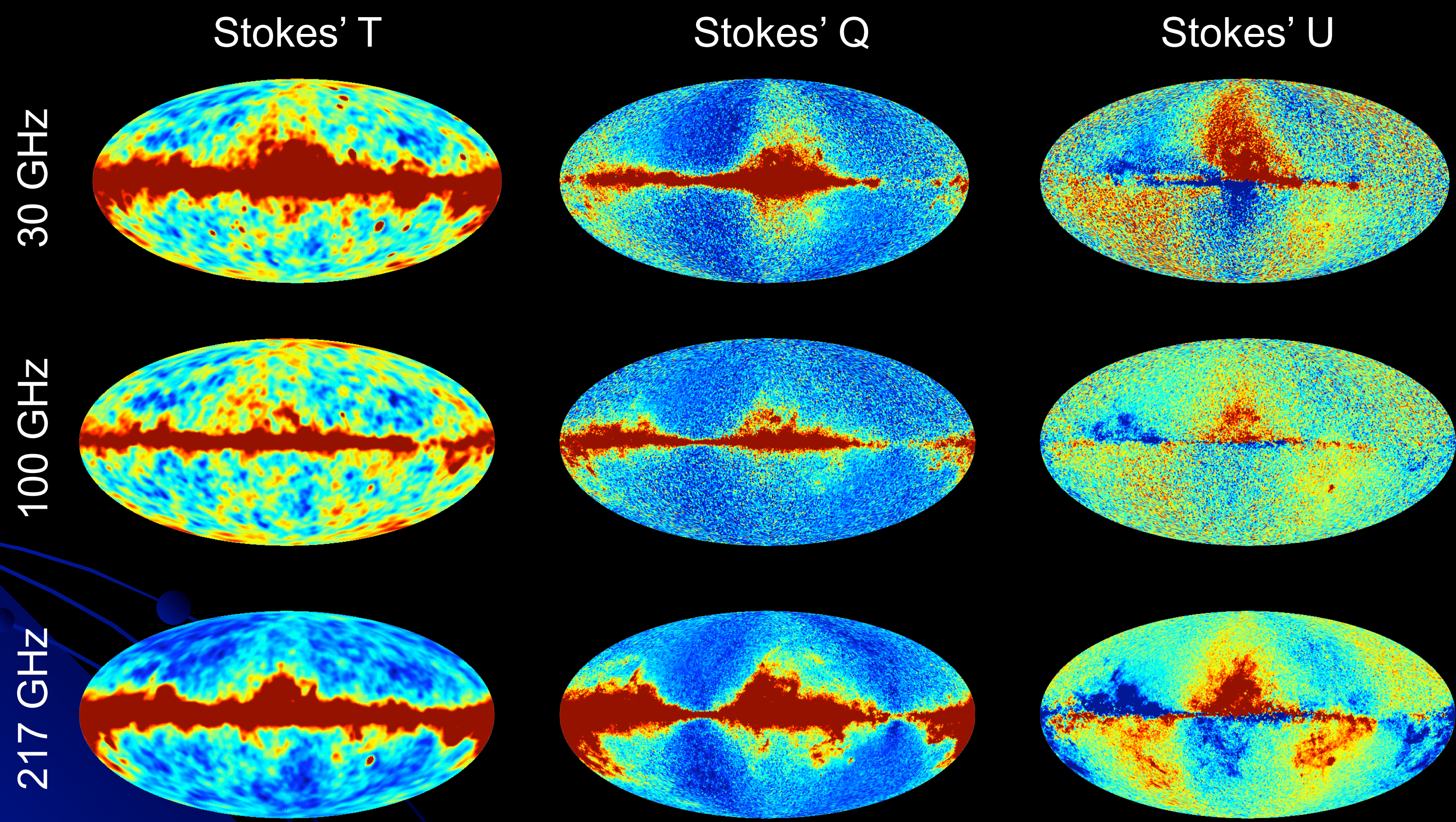


- All maps downgraded to Nside = 64 (55' pixels) to enhance S/N

Simulated sky maps



- All maps downgraded to Nside = 64 (55' pixels) to enhance S/N
- Temperature maps additionally smoothed to 3° FWHM by post-processing, to ensure proper bandwidth limitation

The data model

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- The parametric model adopted for this analysis was the following:

$$S_\nu(p) = \mathbf{s}(p) + m_\nu + \sum_{i=1}^{3} d_i(\hat{p} \cdot \mathbf{e}_i) +$$
$$+ A_s(p)a(\nu)\left(\frac{\nu}{\nu_0}\right)^{\beta_s(p)} + A_d(p)a(\nu)\left(\frac{\nu}{\nu_0}\right)^{\beta_d(p)}$$

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Low-frequency foreground

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Low-frequency foreground

Thermal dust

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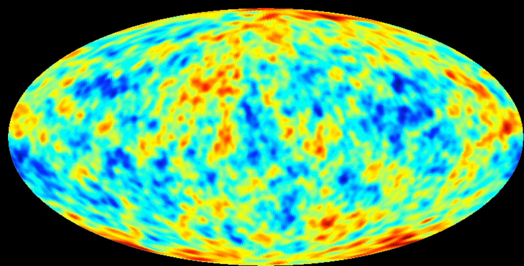
Reconstructed CMB signal maps

Reconstructed CMB signal maps

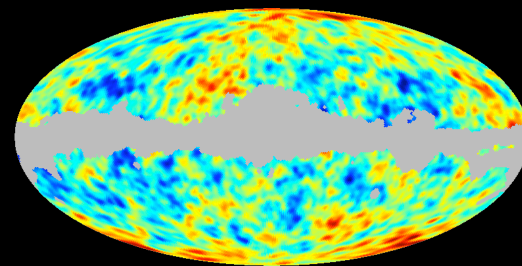
Input signal

Estimated signal

Stokes' T



Scale: -175 μK to 175μK



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Reconstructed CMB signal maps

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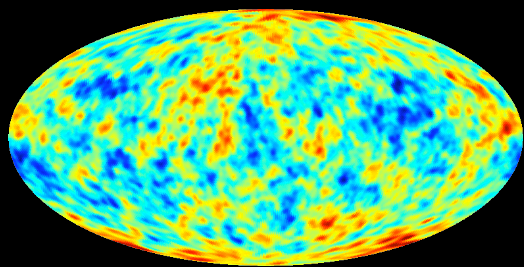

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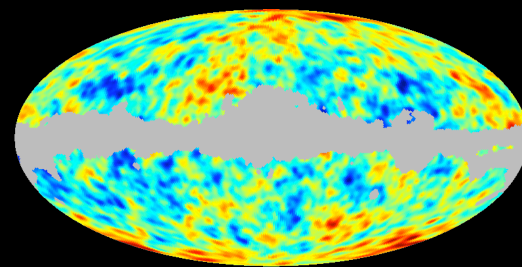

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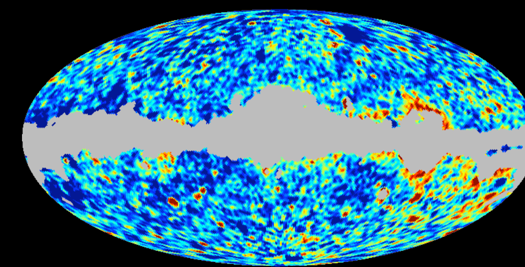
$\frac{\text{Estimate} - \text{Input}}{\text{Estimated error}}$

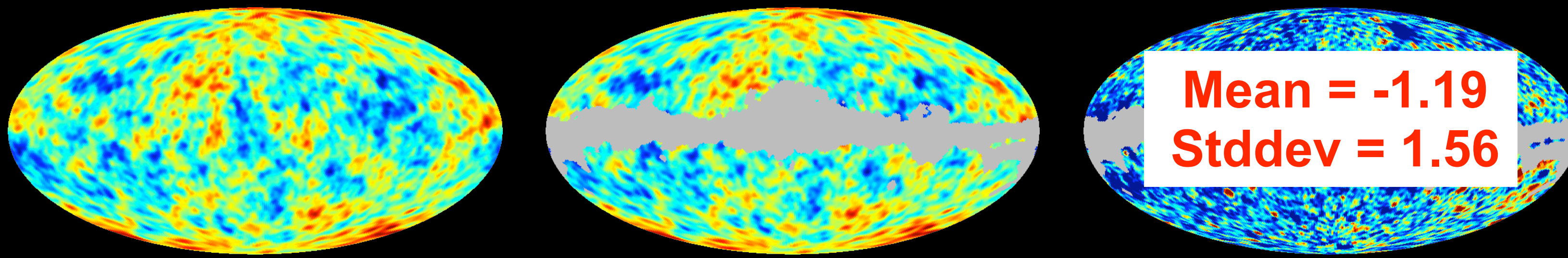


Scale: -3σ to 3σ

Reconstructed CMB signal maps

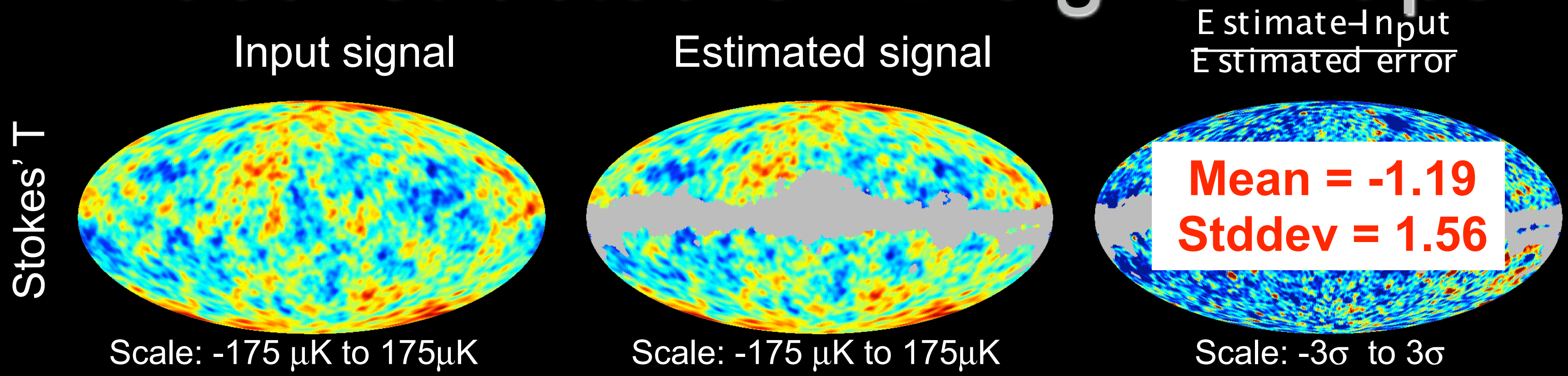
Input signal | Estimated signal | $\frac{\text{Estimate} - \text{Input}}{\text{Estimated error}}$

Stokes' T



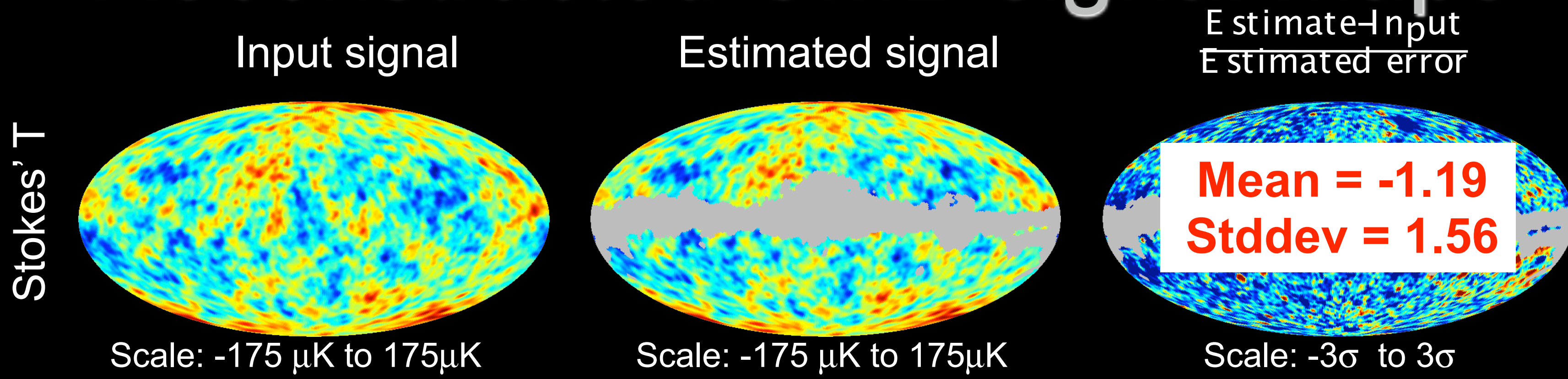
Scale: -175 μK to 175μK | Scale: -175 μK to 175μK | Scale: -3σ to 3σ

Reconstructed CMB signal maps



- Temperature is statistically "difficult" because of (1) multiple low-frequency components and (2) very high S/N

Reconstructed CMB signal maps



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- However, these small (~5-10 μK) temperature errors don't matter to the overall large-scale likelihood because of tremendous cosmic variance

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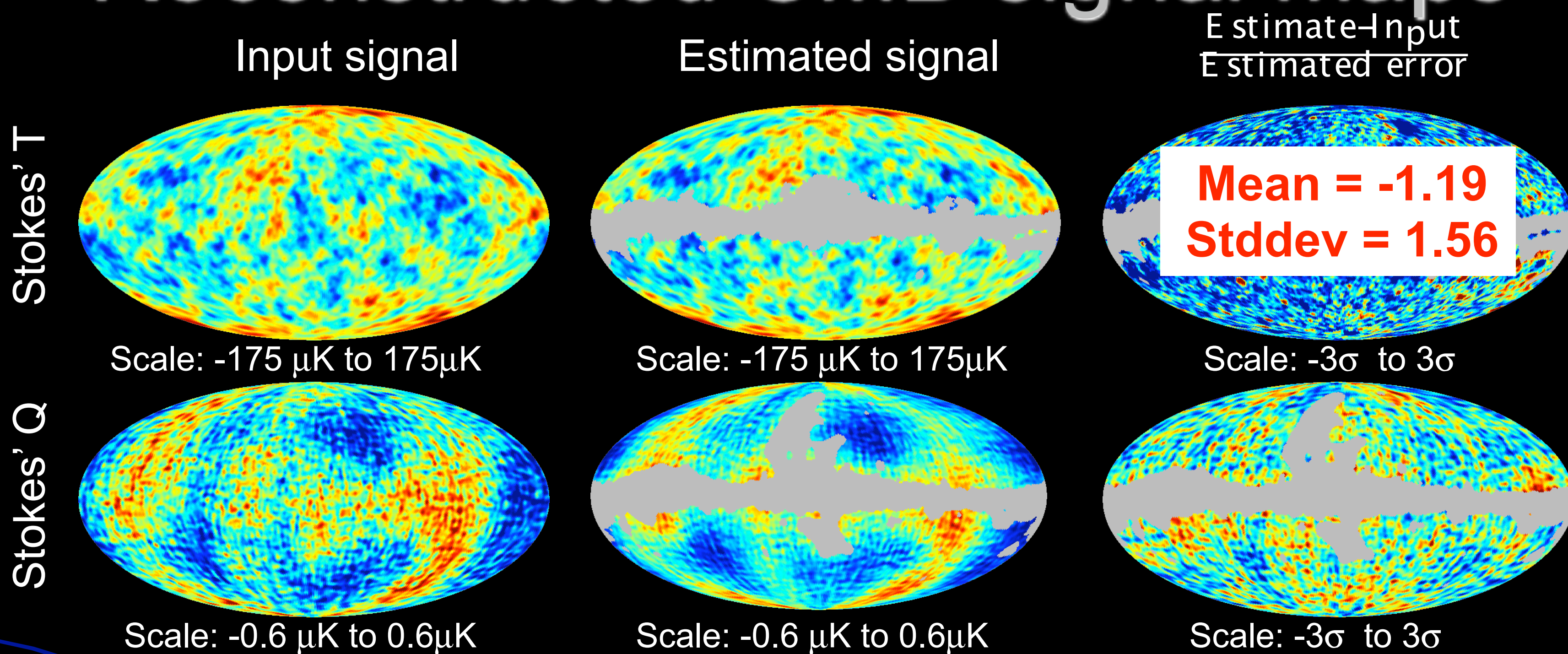

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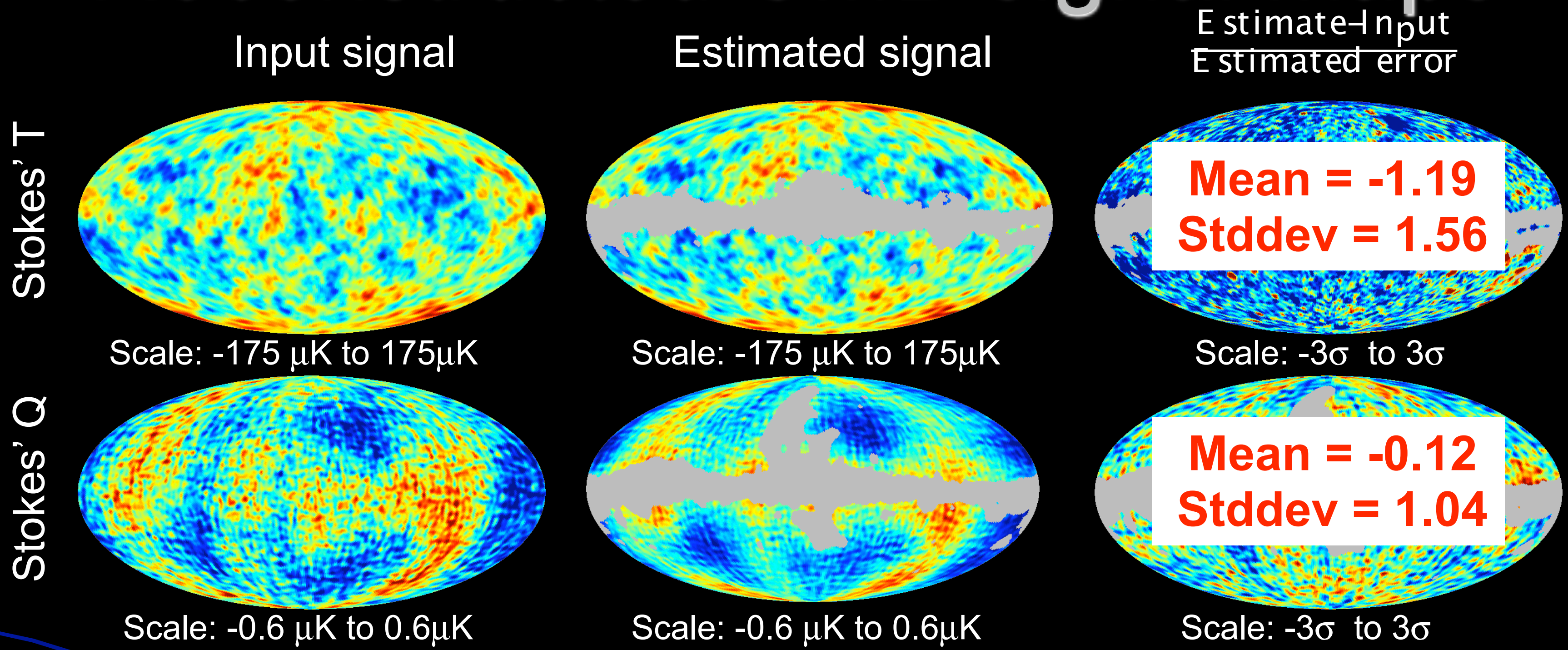

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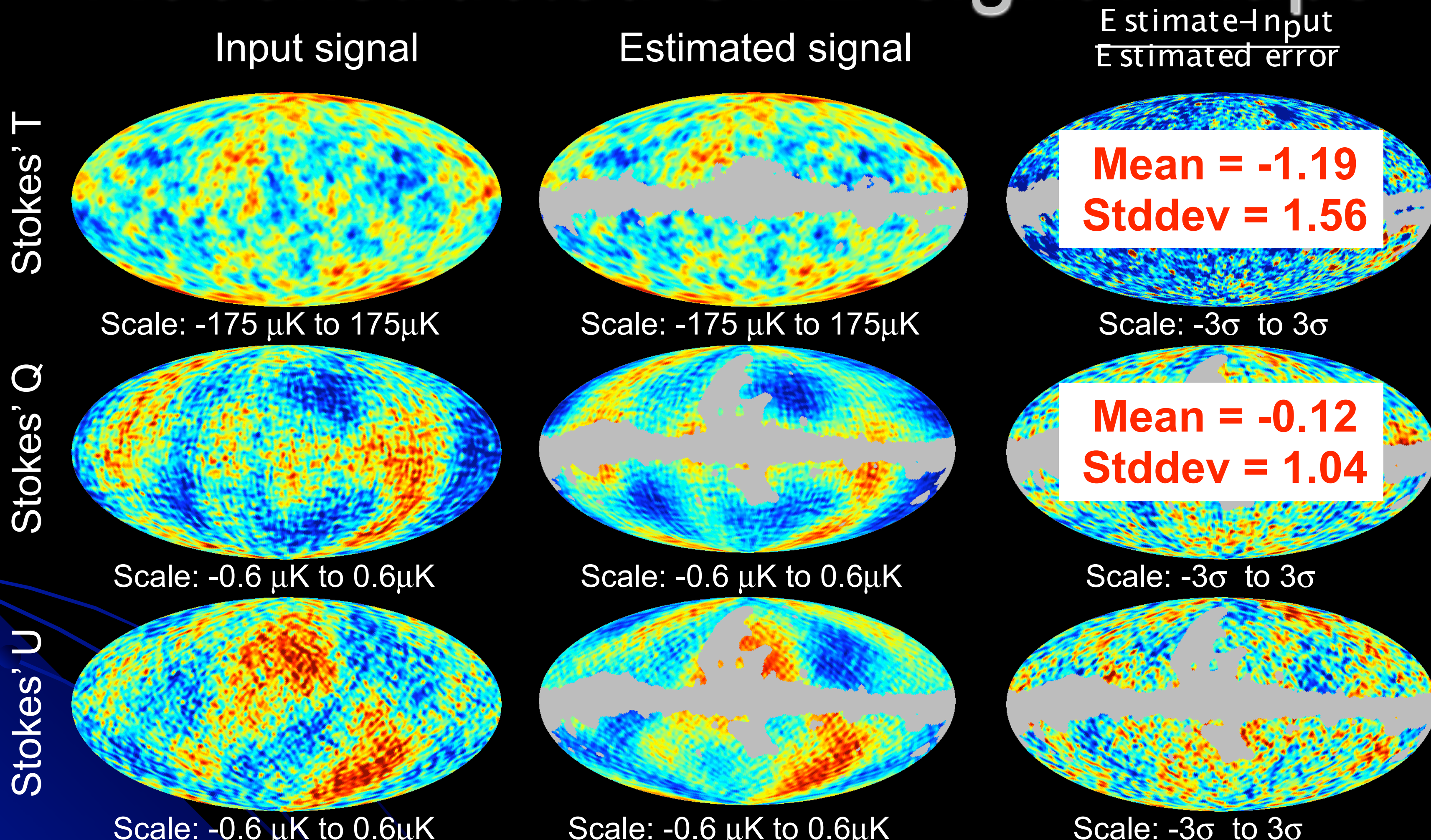

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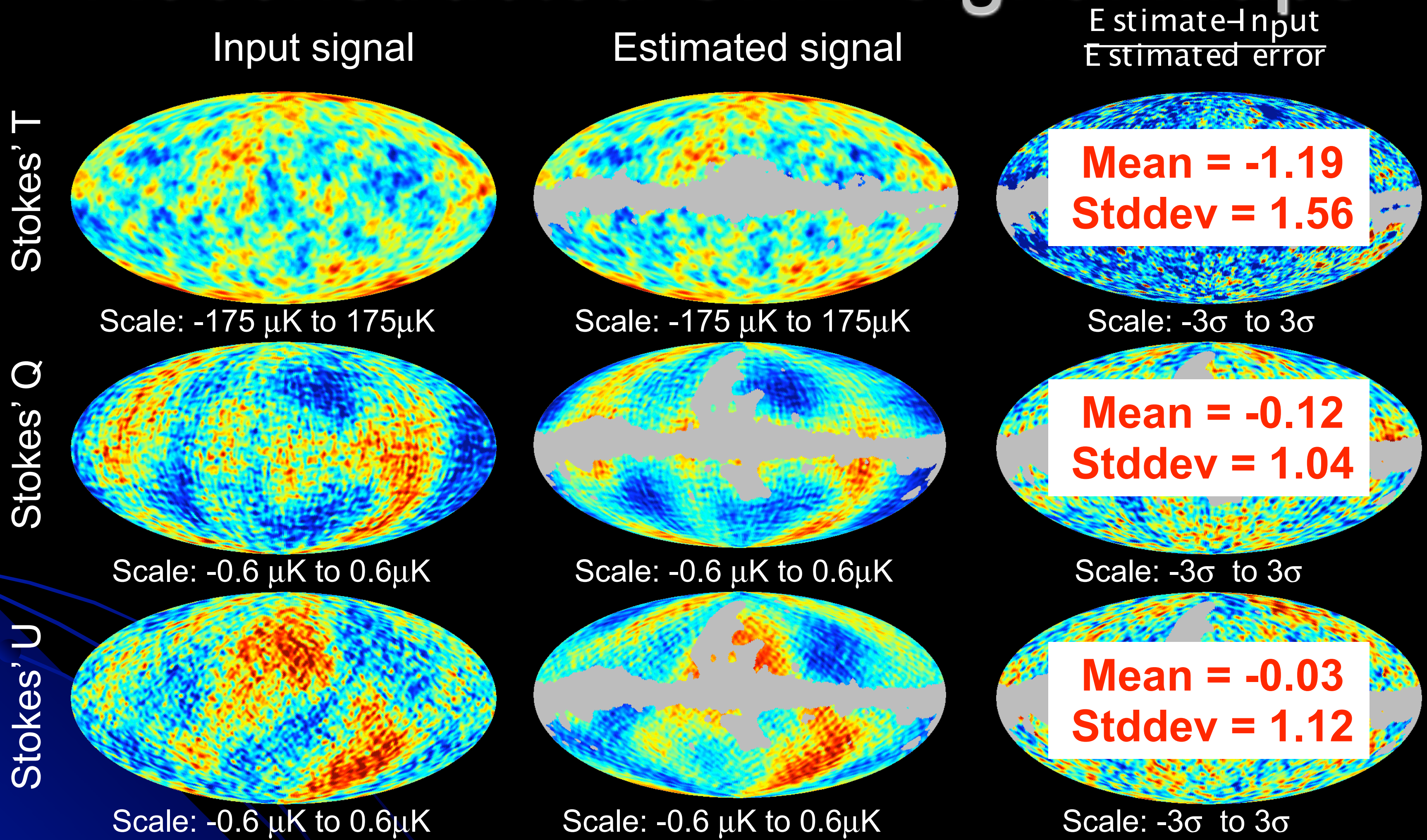

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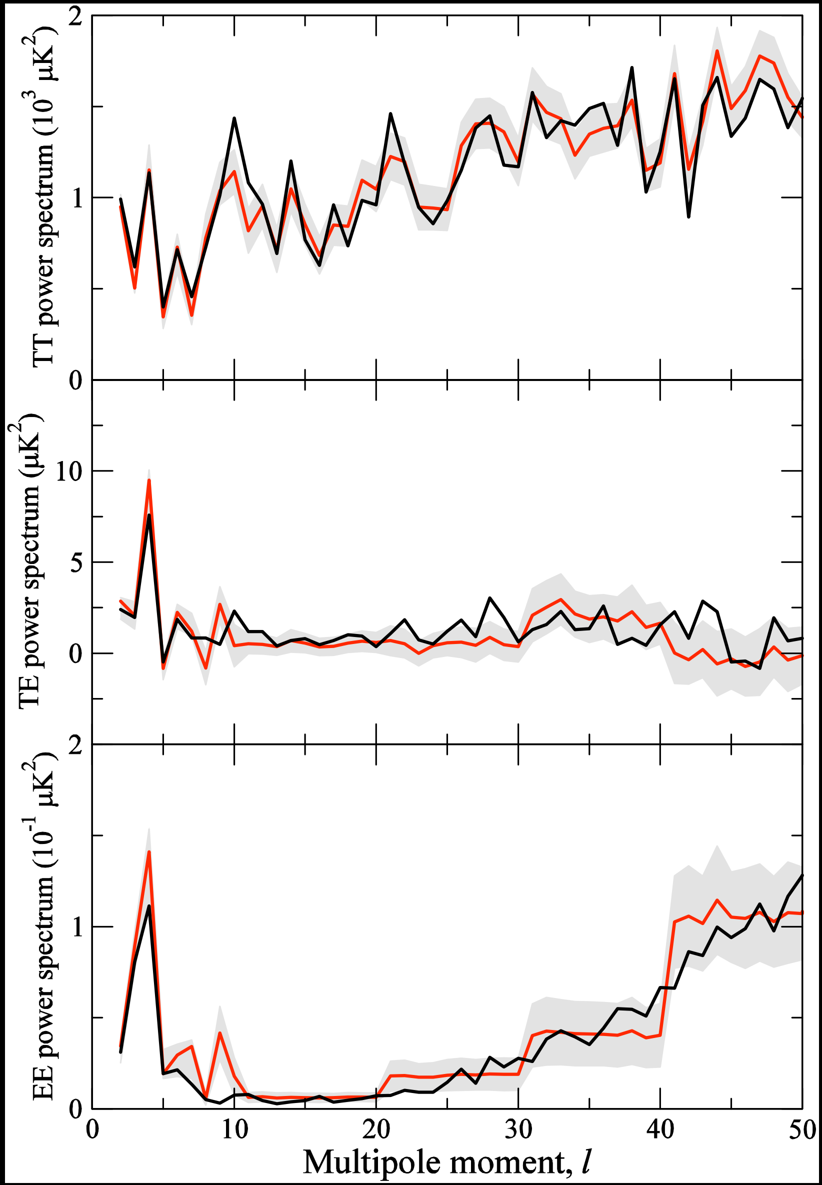

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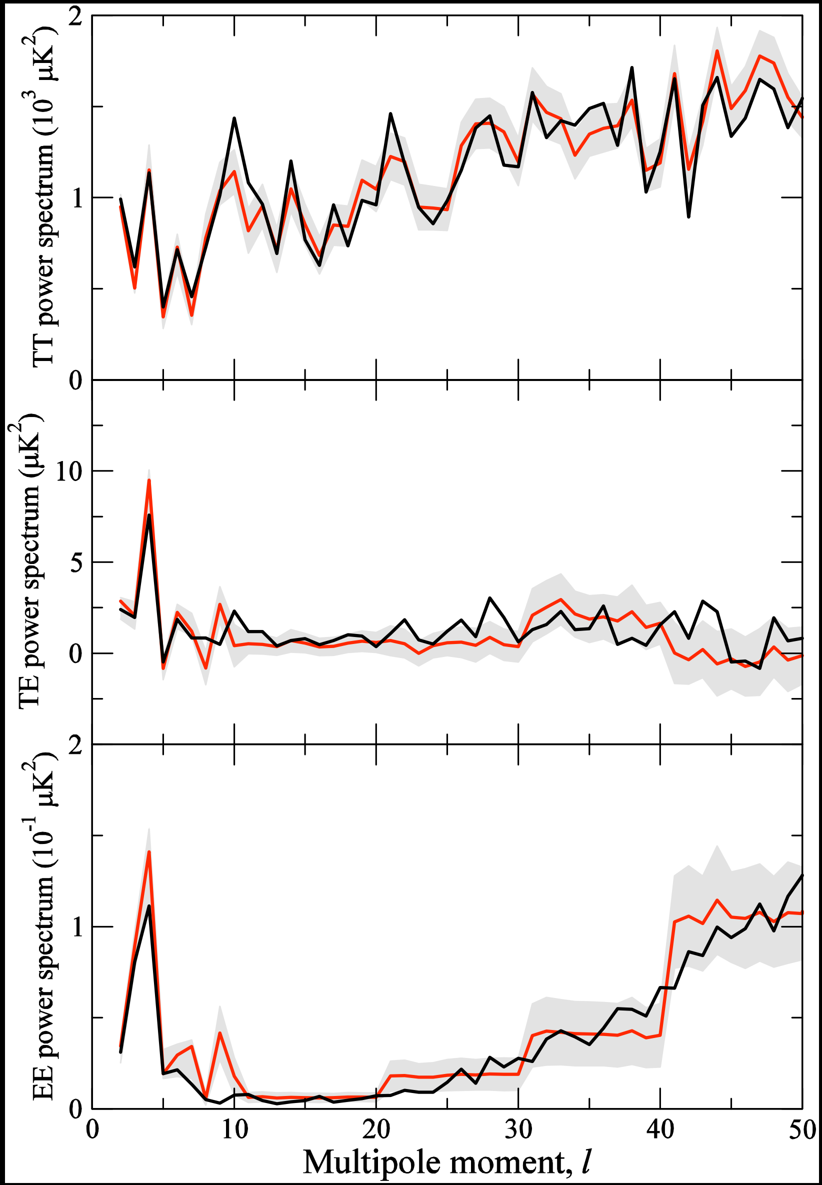

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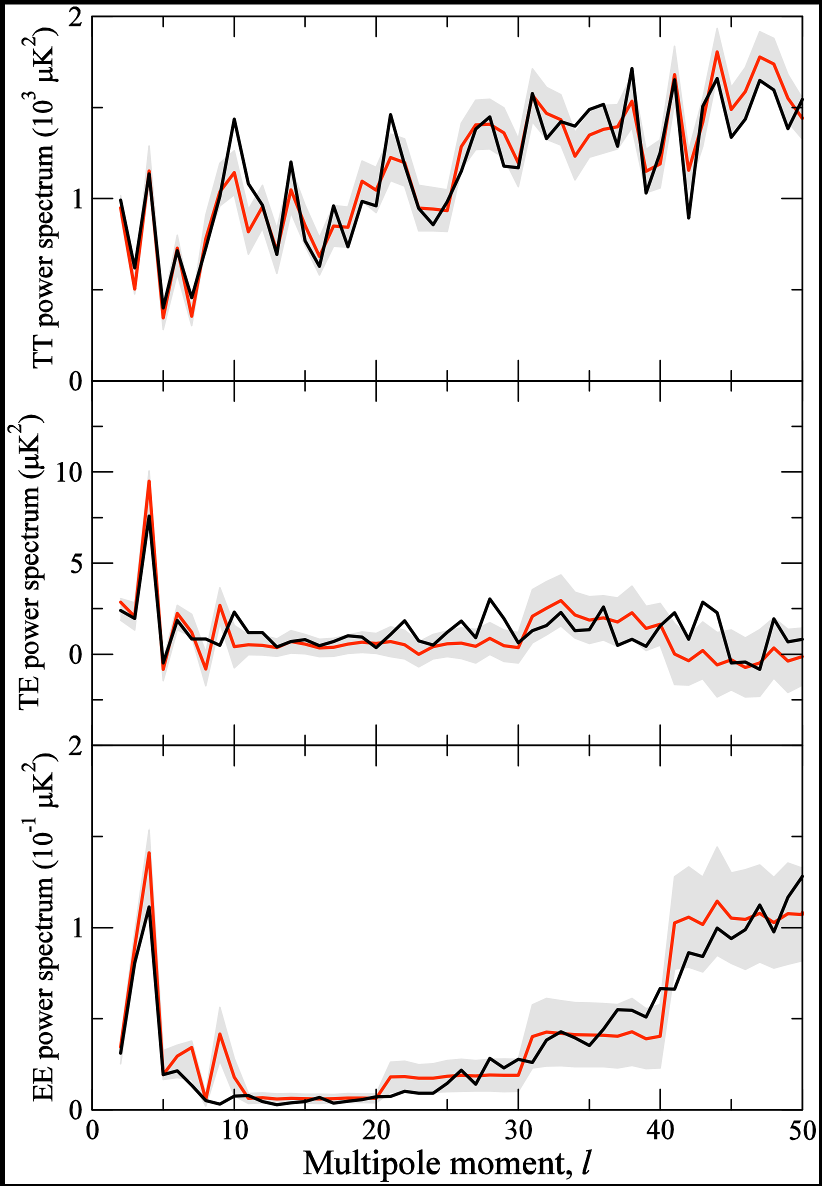

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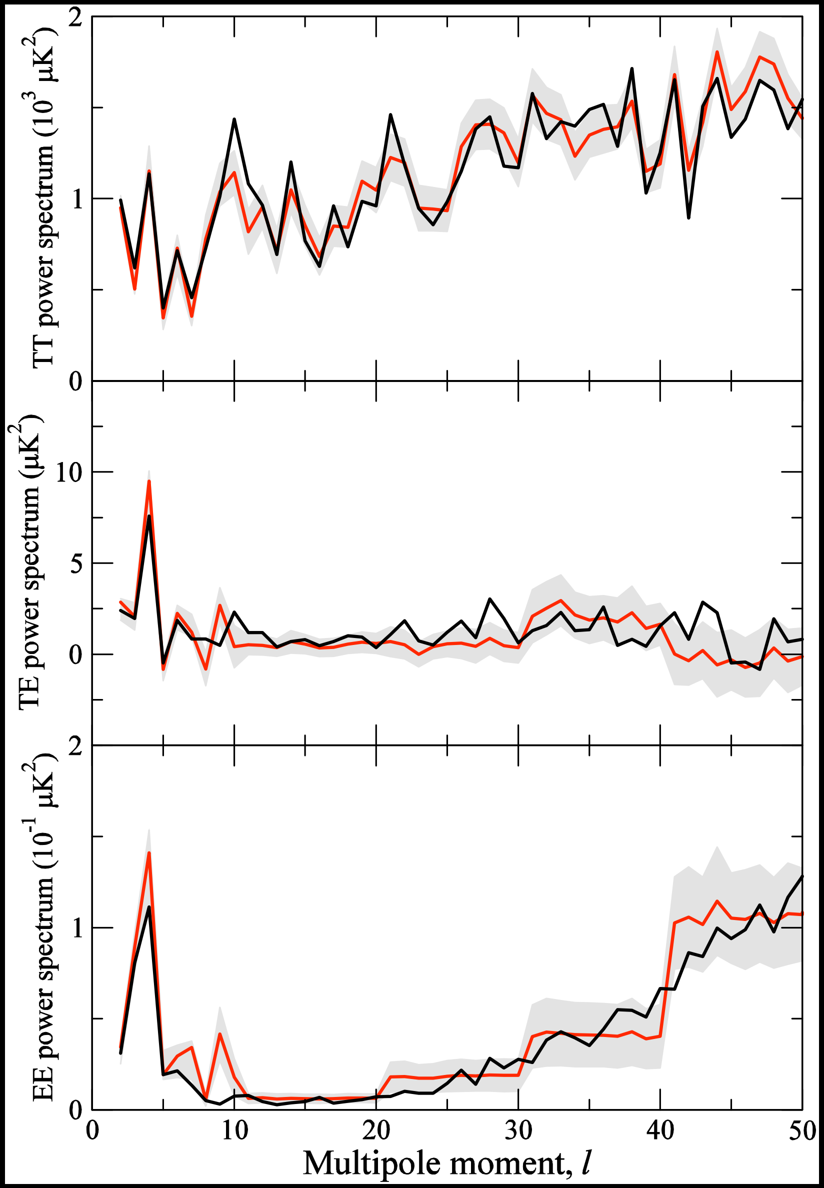

Reconstructed power spectrum

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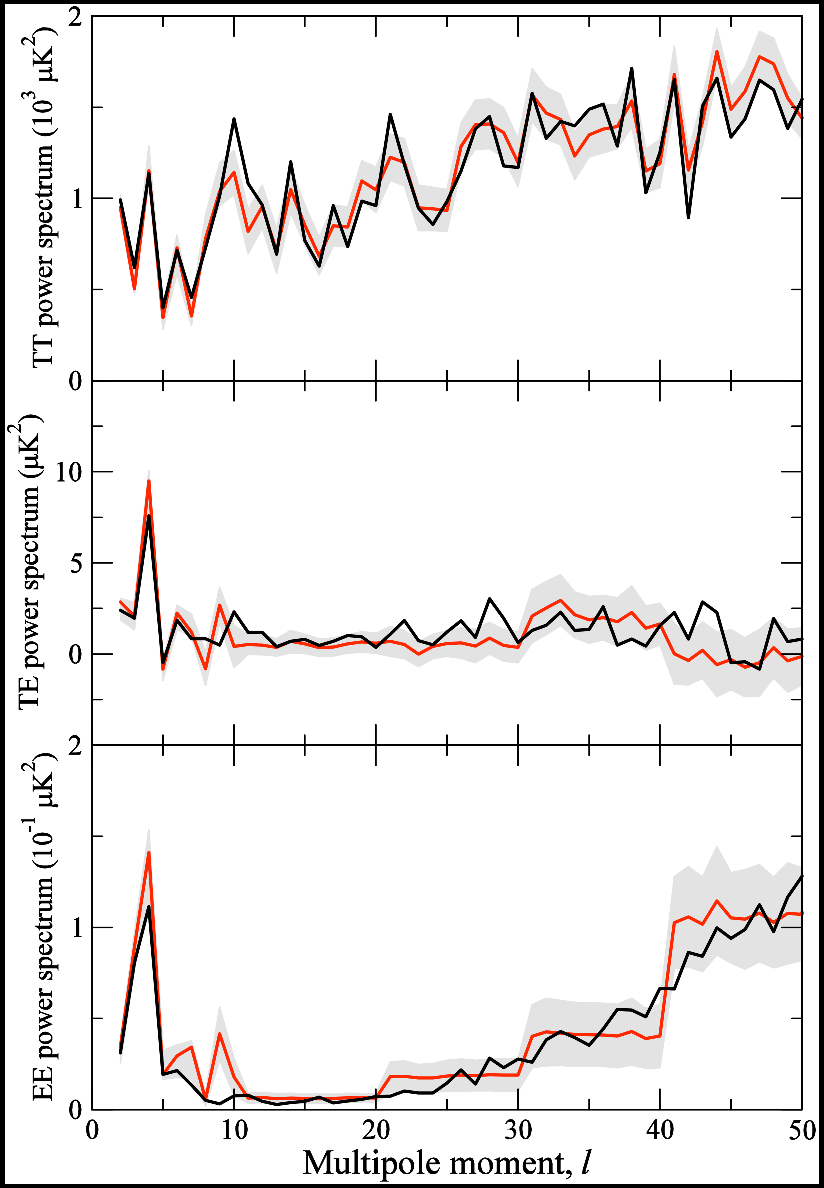
- No striking biases, and error bars are reasonable



CMBPol simulations

CMBPol simulations

- Simulations made by Jo Dunkley:

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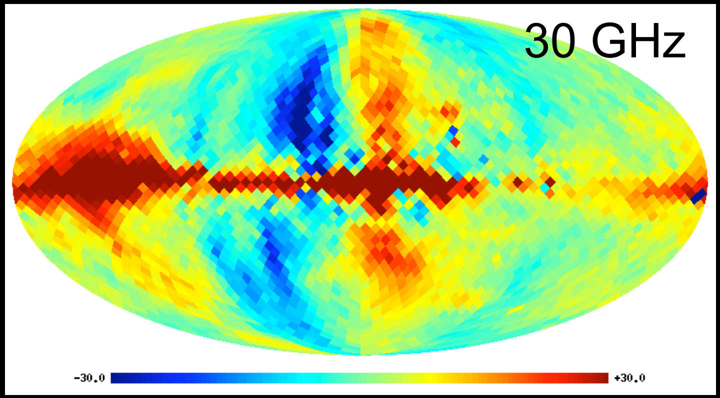

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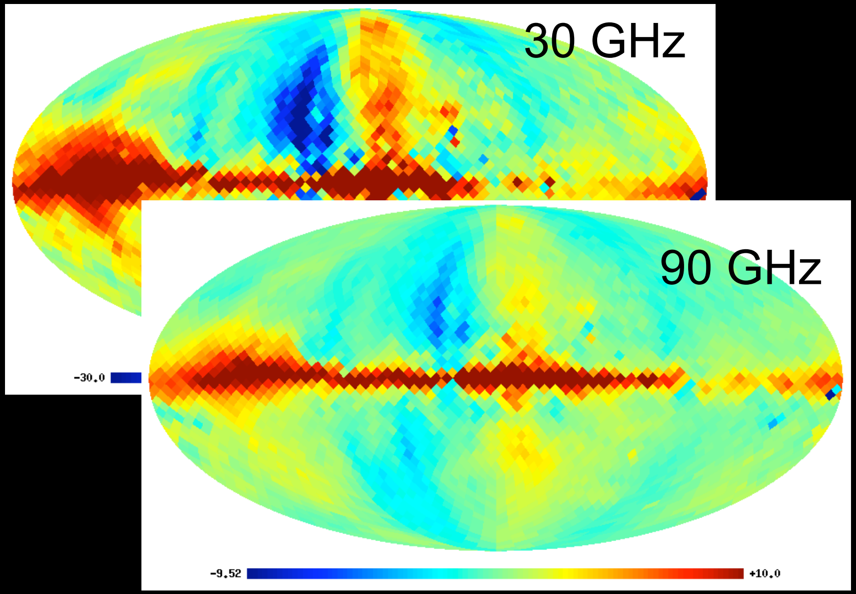

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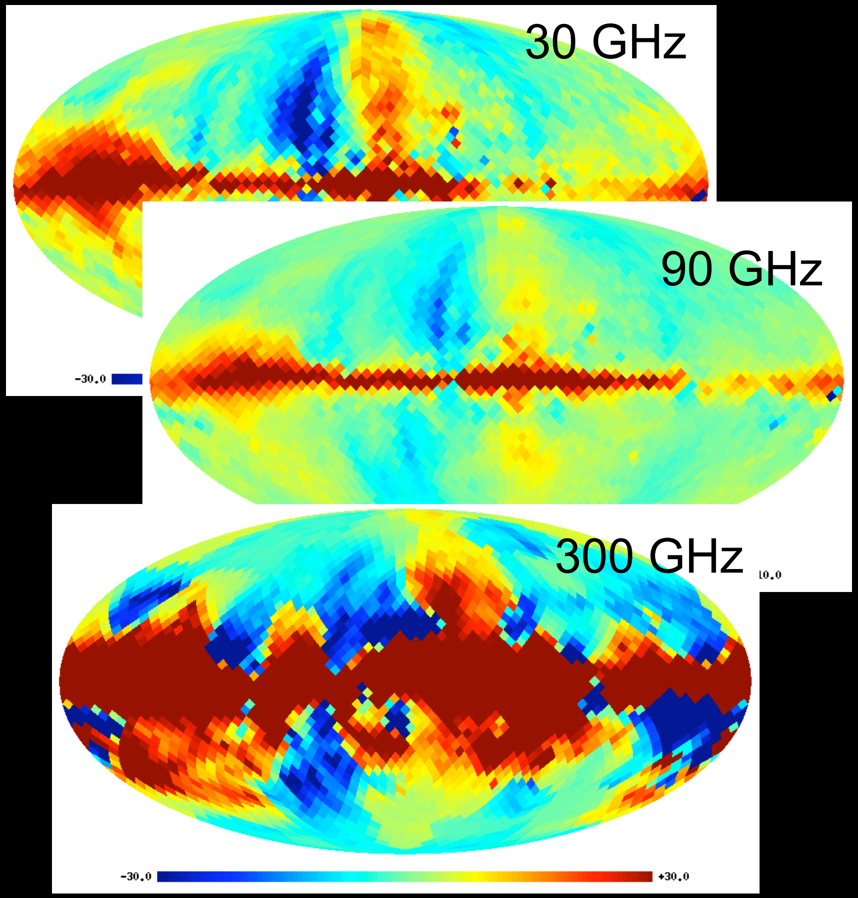

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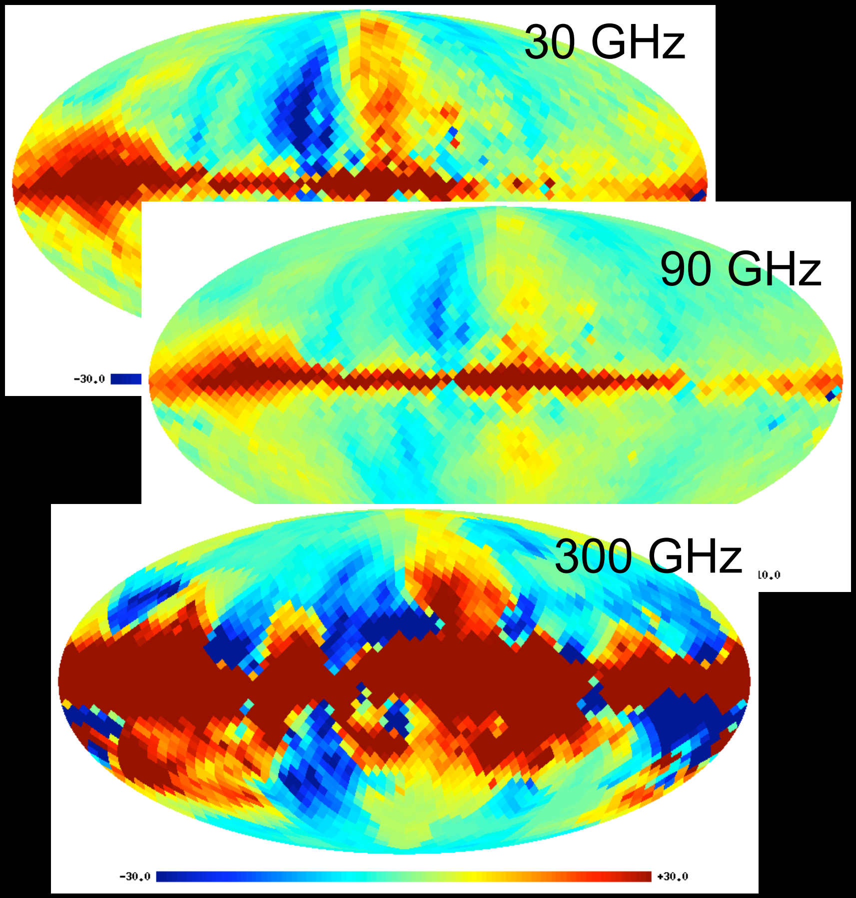

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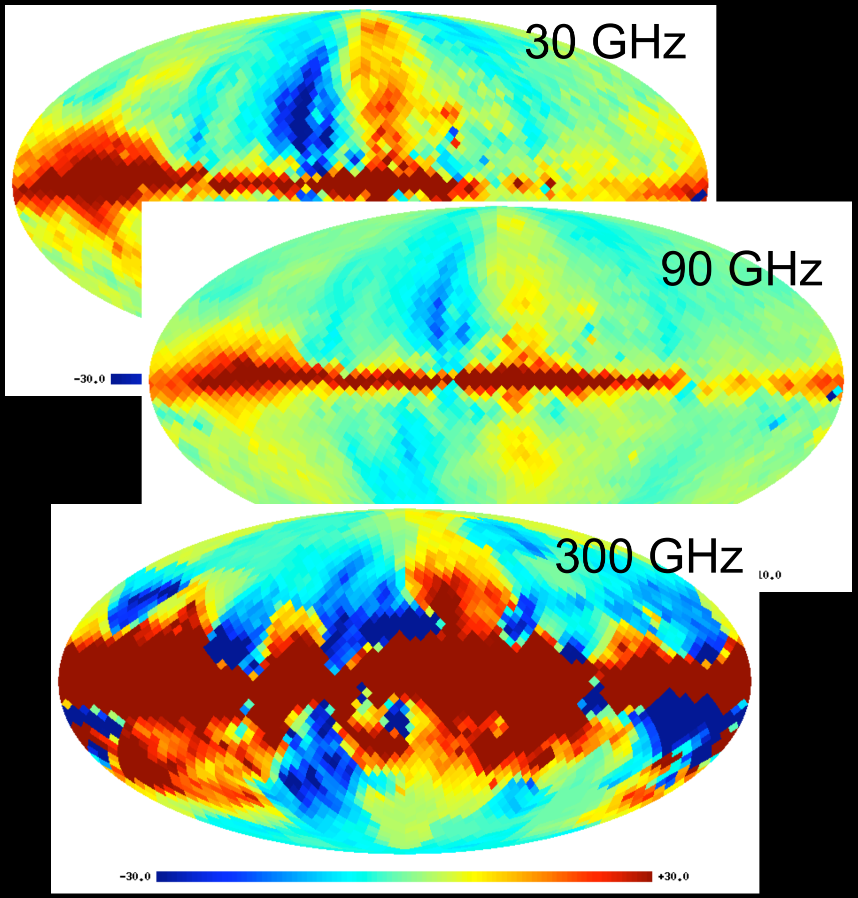

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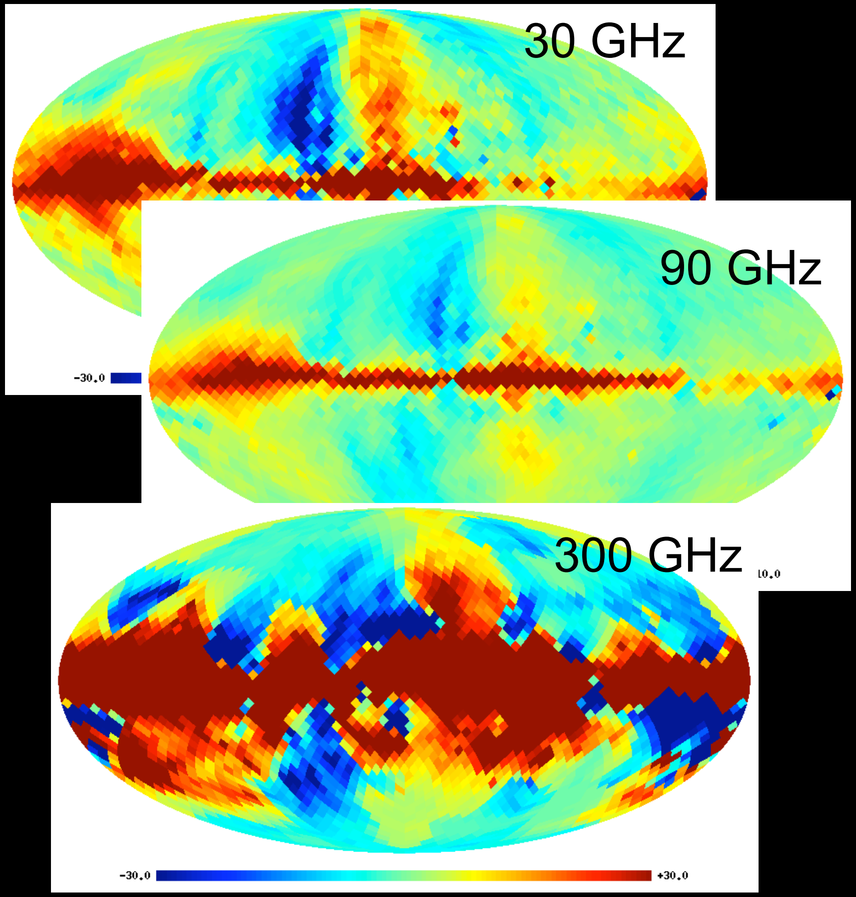

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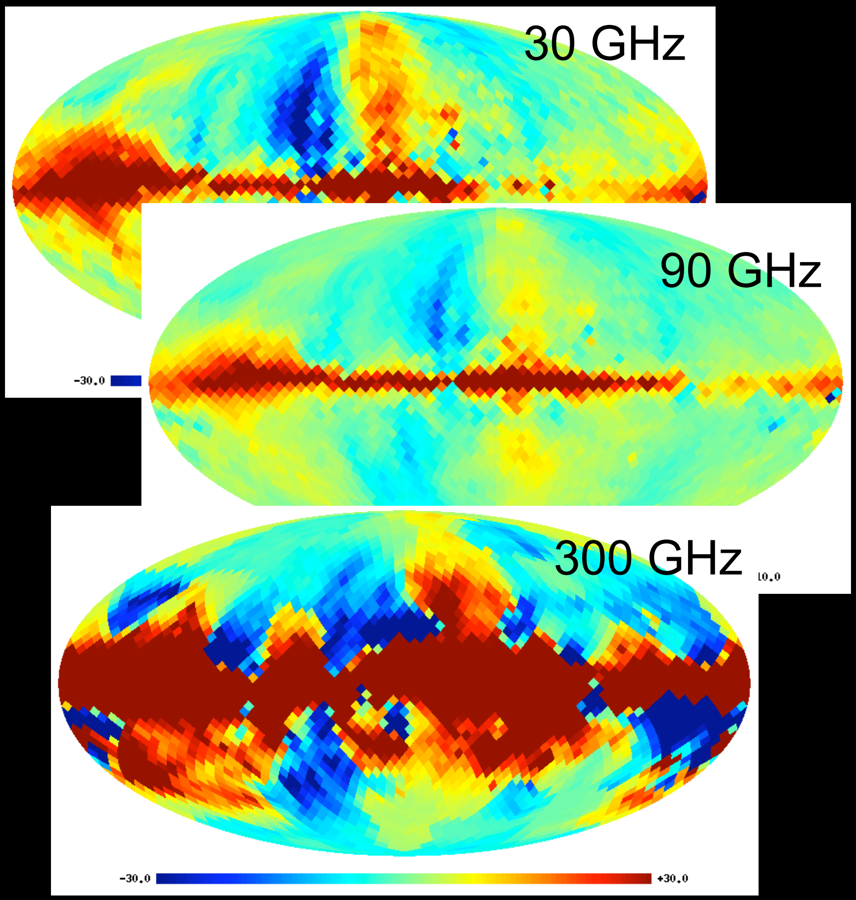

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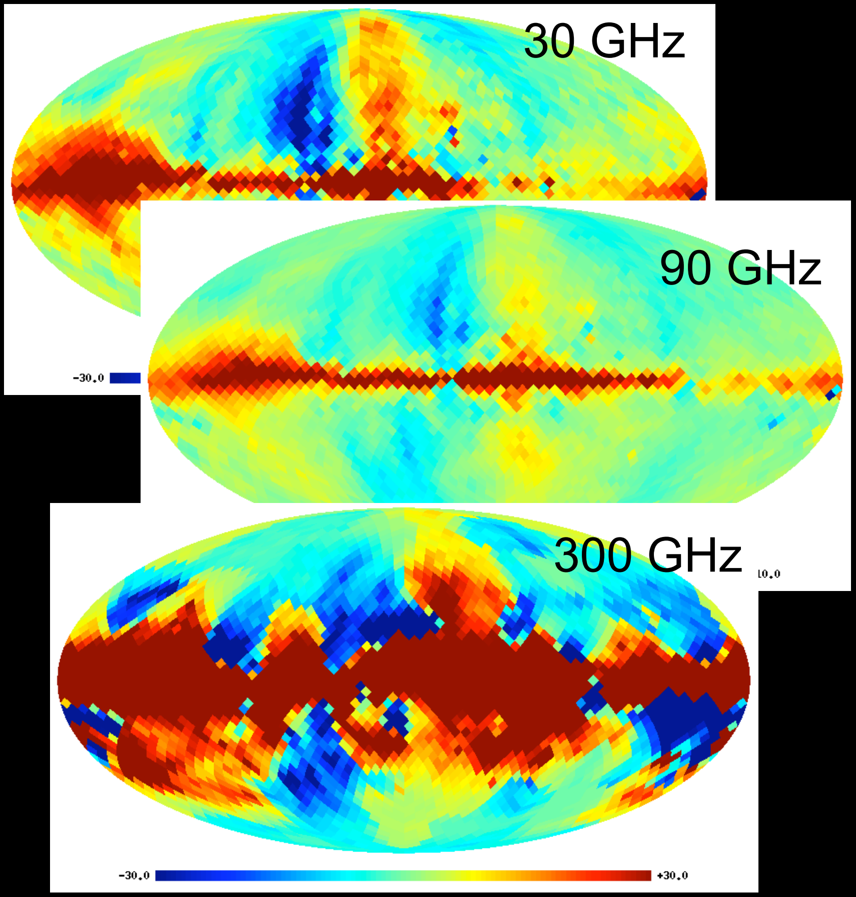

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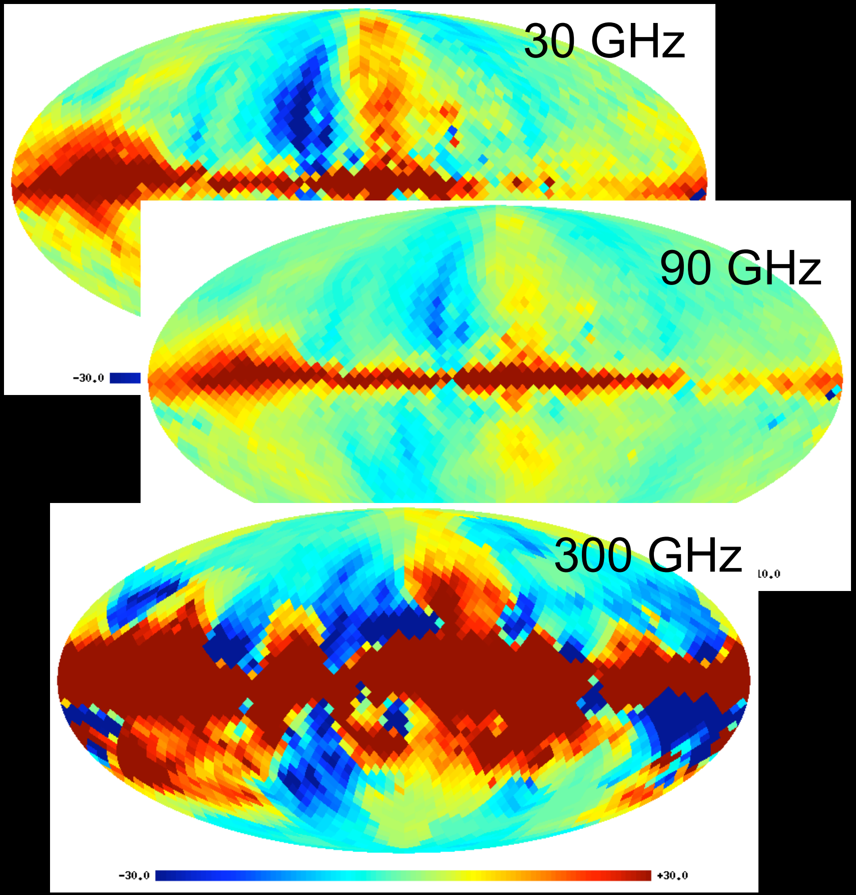
- Analysis at Nside=16, lmax=47, FWHM = 7 degrees, to speed up things during test phase



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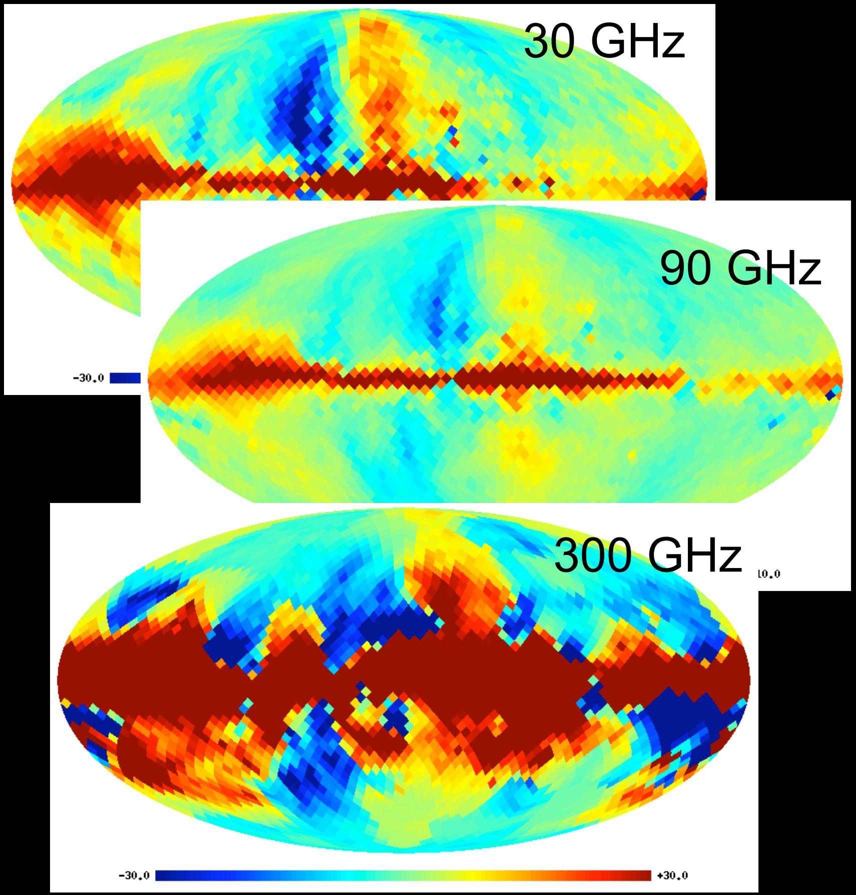

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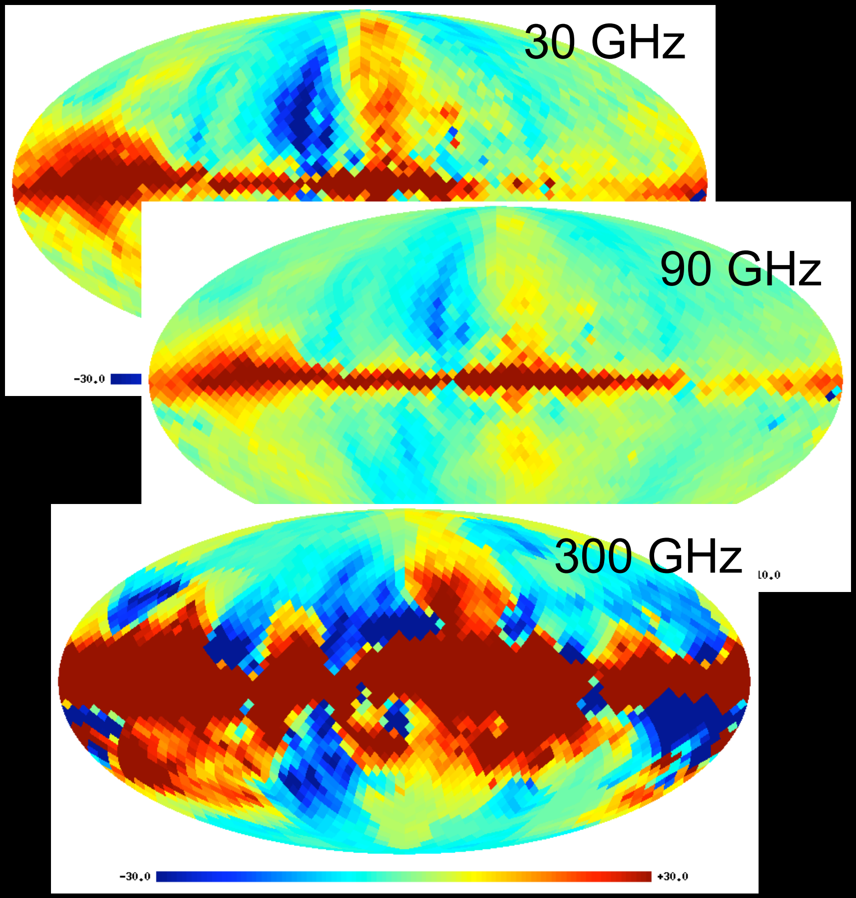
- (Note: These results are a first test, hastily computed last week, and intended only to demonstrate the machinery)



Foreground model and free parameters

Foreground model and free parameters

- Simplest possible model adopted:

$$S_o(p) = s(p) + A_s\, g(o)\, 3 \left(\frac{o}{30\,\mathrm{GHz}}\right)^{i\,3} + A_d\, g(o)\, 3 \left(\frac{o}{300\,\mathrm{GHz}}\right)^{1:7}$$

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  - Only marginalization over foreground amplitudes, not spectral indices

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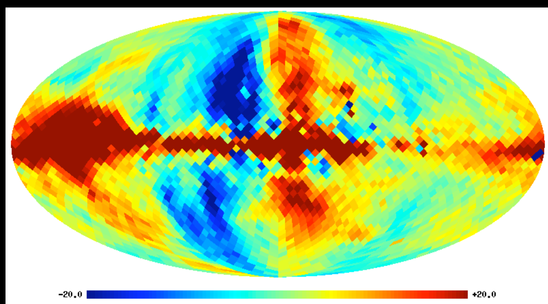
  - But not *really* what we want, since different from standard (primordial based) *r*

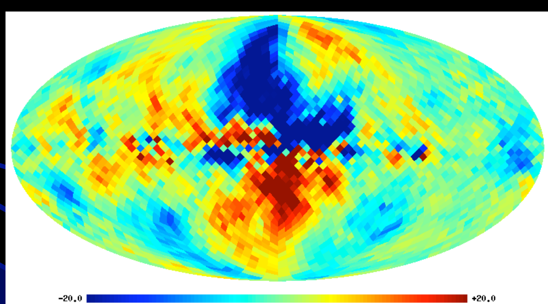
Reconstructed sky maps

Posterior mean

Stokes' Q



Stokes' U



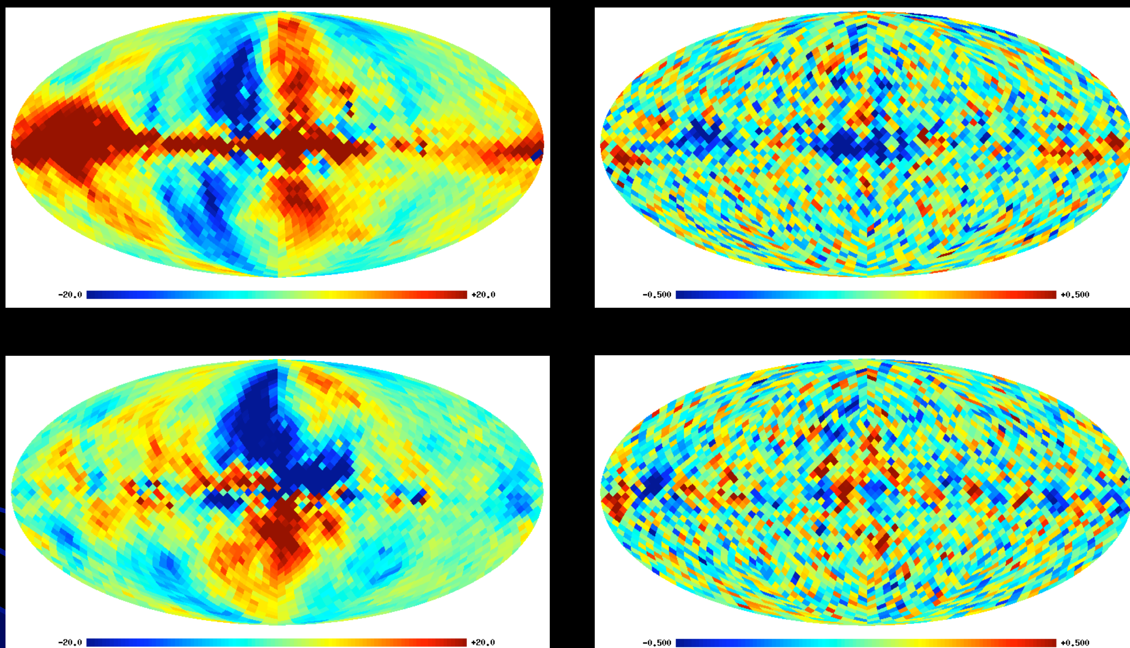
Scale: -20 to 20 μK

Reconstructed sky maps

Posterior mean　　Mean - Input

Stokes' Q

Stokes' U



Scale: -20 to 20 μK

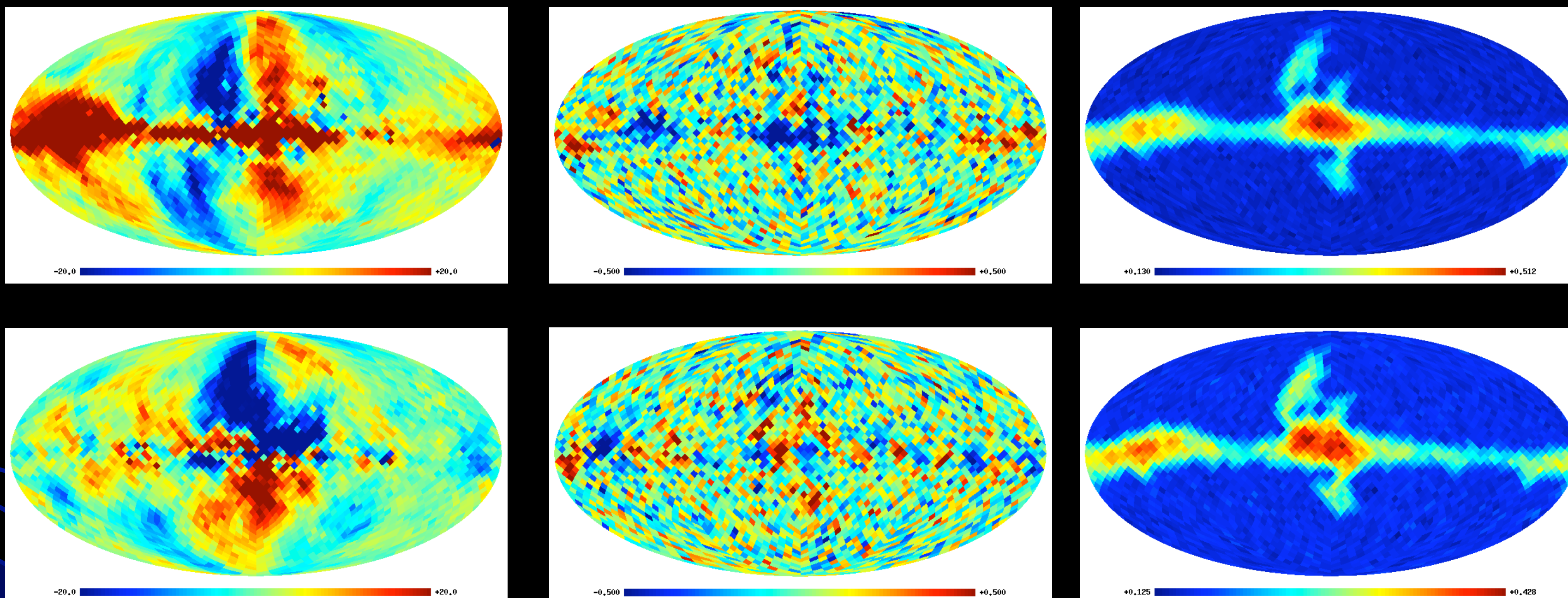
Scale: -0.5 to 0.5 μK

Reconstructed sky maps

Posterior mean | Mean - Input | Posterior RMS

Stokes' Q

Stokes' U



Scale: -20 to 20 μK

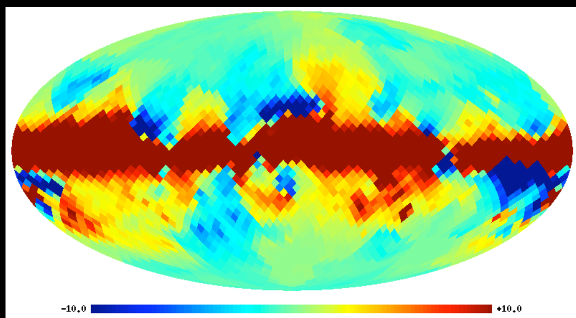
Scale: -0.5 to 0.5 μK

Scale: 0.13 to 0.5 μK

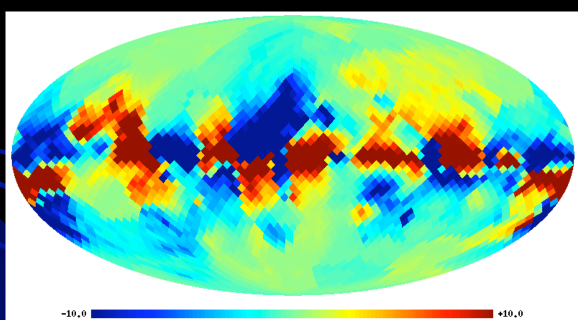
Reconstructed dust maps

Posterior mean

Stokes' Q



Stokes' U



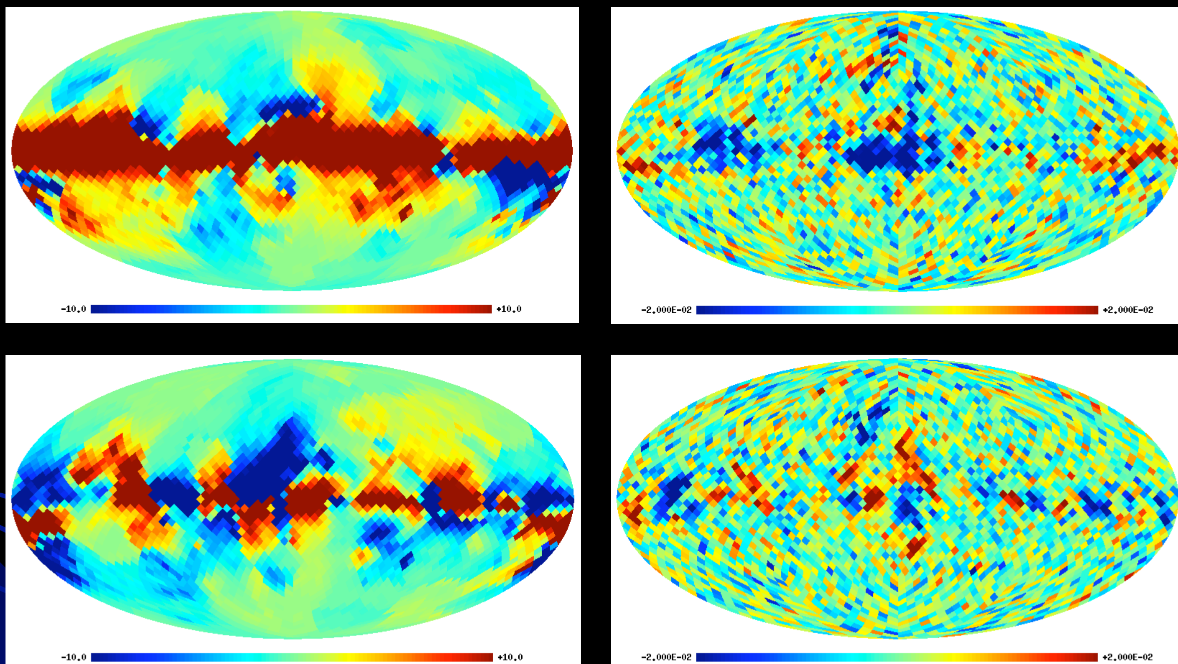
Scale: -10 to 10 μK

Reconstructed dust maps

Posterior mean | Mean - Input

Stokes' Q

Stokes' U



Scale: -10 to 10 μK

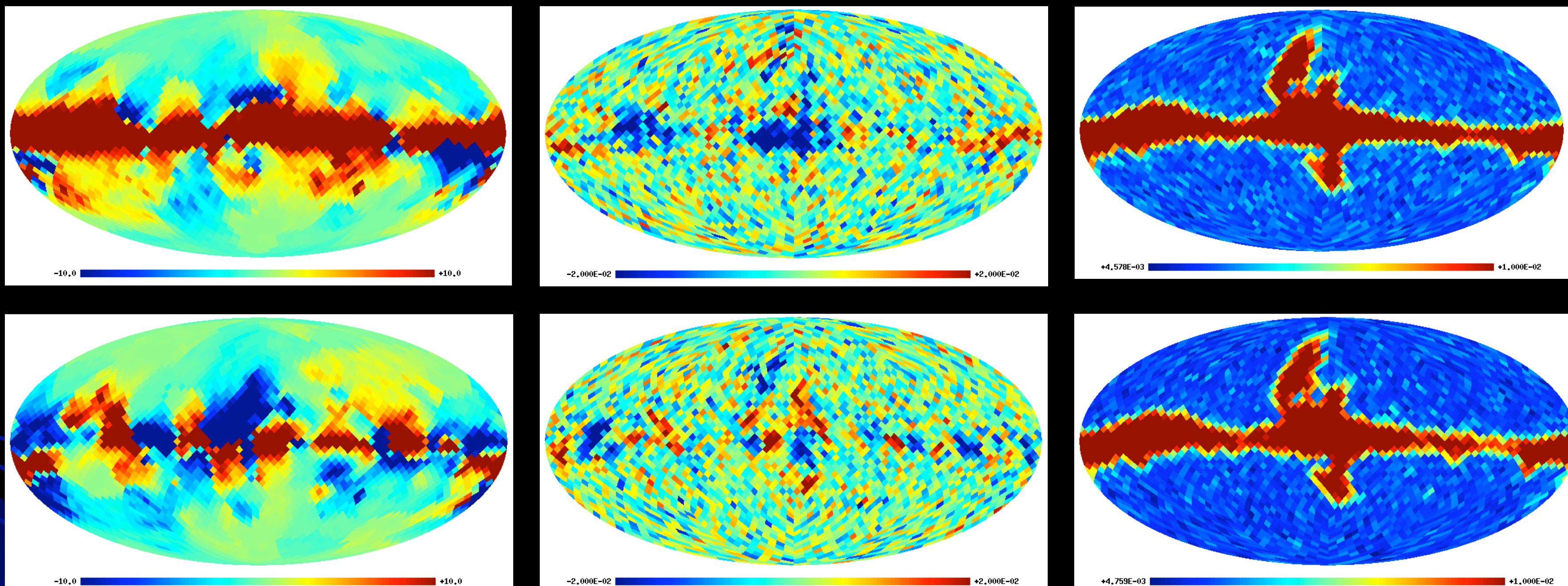
Scale: -0.02 to 0.02 μK

Reconstructed dust maps

Posterior mean | Mean - Input | Posterior RMS

Stokes' Q

Stokes' U

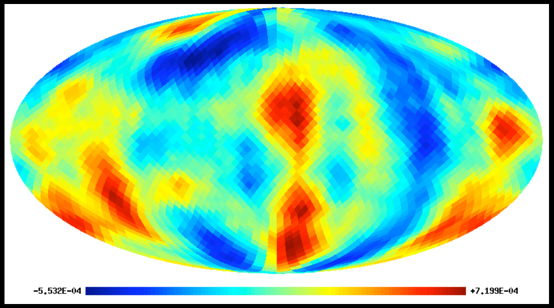


Scale: -10 to 10 μK | Scale: -0.02 to 0.02 μK | Scale: 0.005 to 0.02μK

Reconstructed CMB maps

Posterior mean

Stokes' Q



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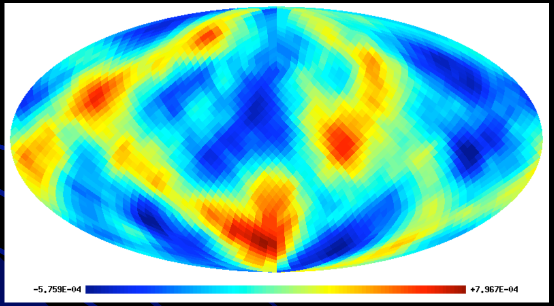
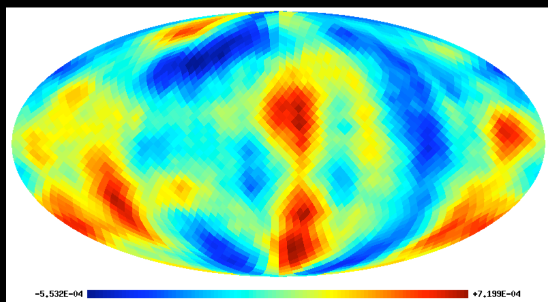


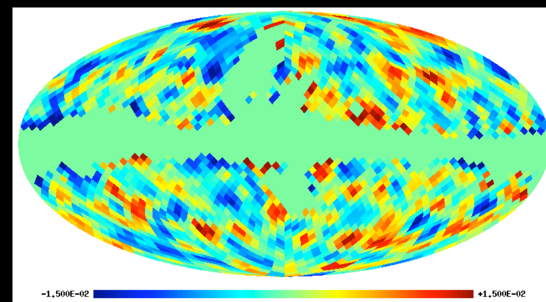
Scale: -0.5 to 0.7 μK

Reconstructed CMB maps

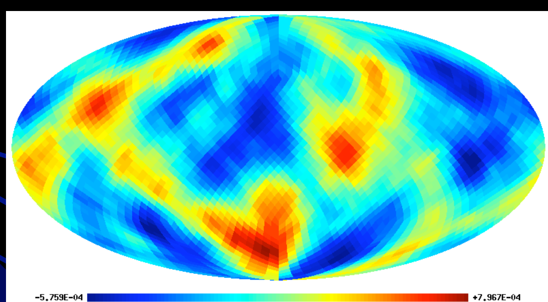
Posterior mean | Mean - Input

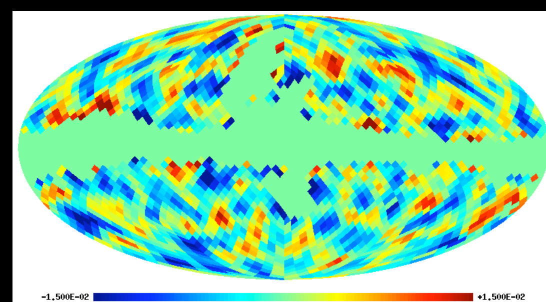
Stokes' Q





Stokes' U





Scale: -0.5 to 0.7 μK

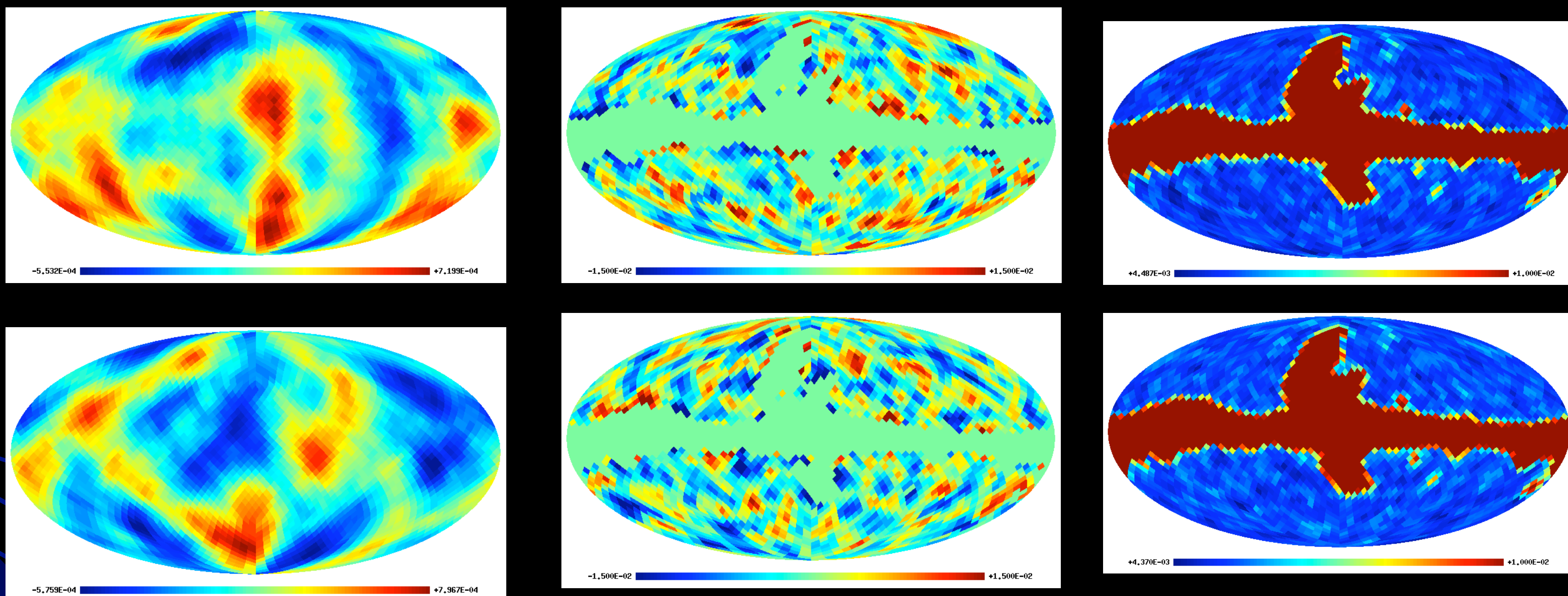
Scale: -0.015 to 0.015 μK

Reconstructed CMB maps

Posterior mean | Mean - Input | Posterior RMS

Stokes' Q

Stokes' U



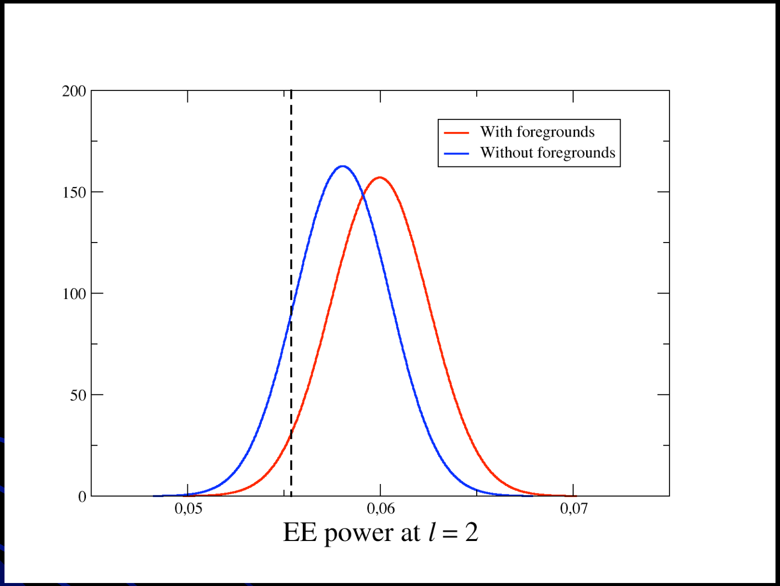
Scale: -0.5 to 0.7 μK

Scale: -0.015 to 0.015 μK

Scale: 0.005 to 0.01 μK

Reconstructed EE and BB power

EE power at l = 2



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- Vertical dashed: True input value

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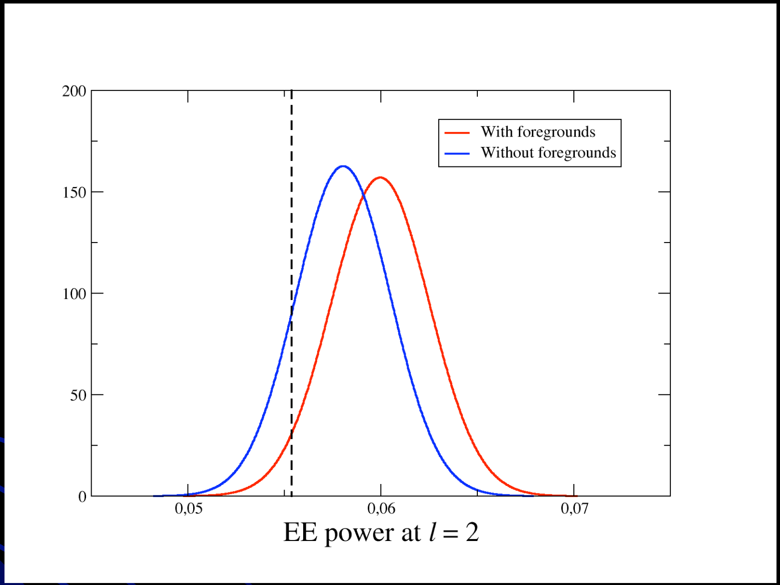
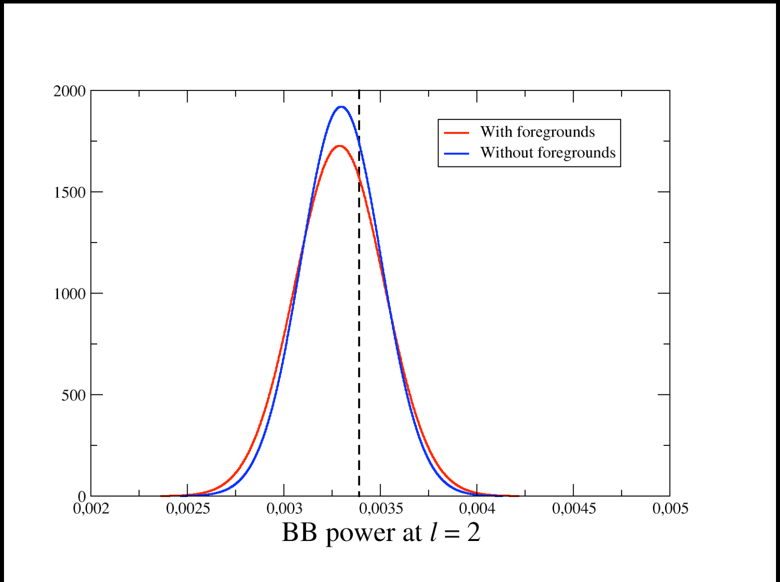


BB power at l = 2



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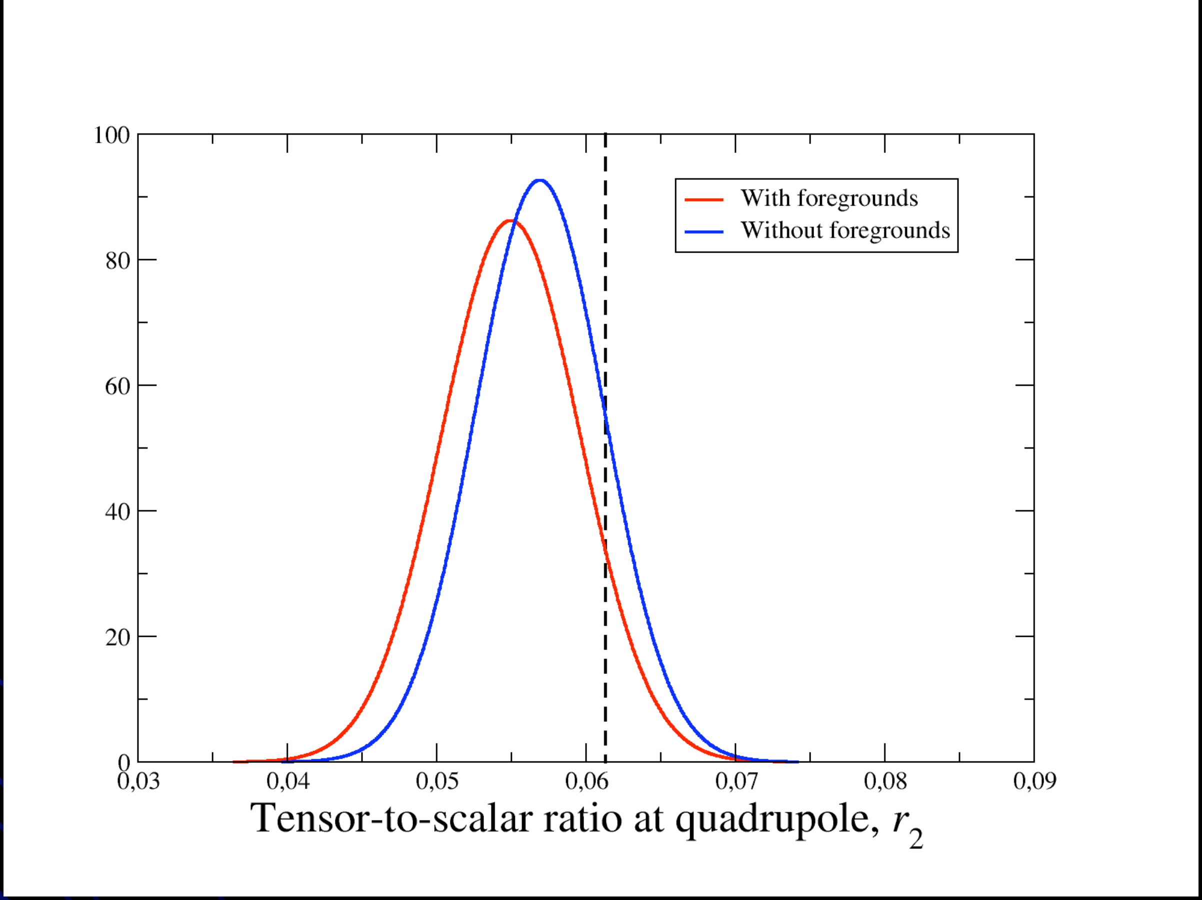
Reconstructed tensor-to-scalar ratio



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LC1; no foregrounds	0.0569	0.0043	13.2	1
LC1; foregrounds	0.0550	0.0046	12.0	1.07
LC2; no foregrounds	0.0609	0.0056	10.9	1.30
LC2; foregrounds	0.0589	0.0062	9.5	1.44

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- Next steps are 1) to marginalize over spectral indices, and 2) go to higher resolution (N_{side} = 128, lmax = 300, 90 arcmin FWHM)

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  - Accuracy is of less importance; speed is essential

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  - Then analyse a few select cases with the full Gibbs machinery to verify the above findings, and demonstrate our general analysis capability