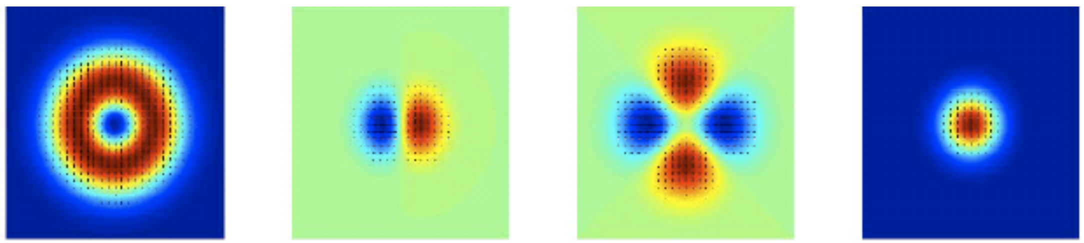
# CMB Systematics and Mitigation Techniques



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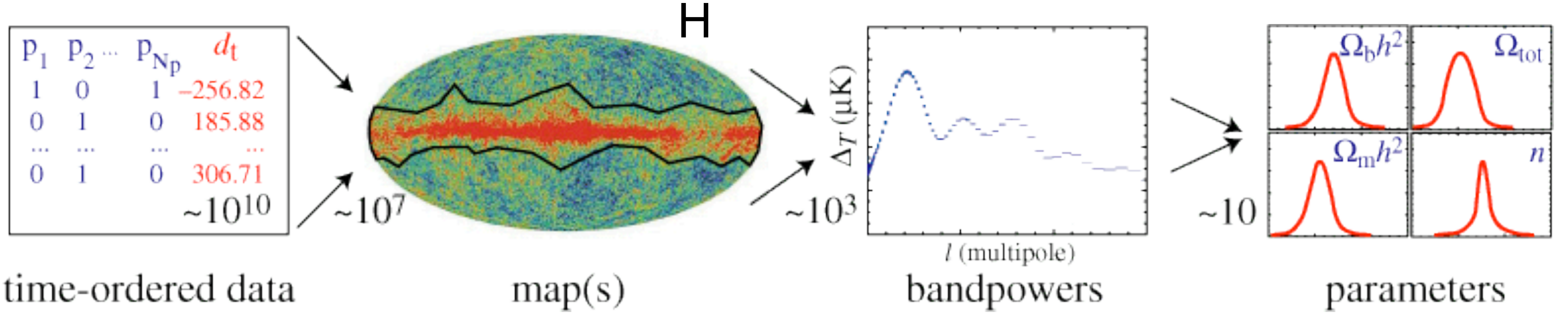
UCSD

# Motivation

- Cosmological model depends on a dozen parameters
- Parameter degeneracy and polarization
- Importance of the B-mode
- Susceptibility to temperature leakage

# From Data Acquisition to Cosmological Parameters

W
H




systematics

# Beam Convolution

- Neglecting the effect of scanning strategy beam systematics depend on the beam and underlying sky only

$$F \text{ ® } F \otimes W, \quad F \in \{T, Q, U\}$$

$$\widetilde{F} \text{ ® } \widetilde{F} \times \widetilde{W}$$

- Therefore, the statistical nature of *F* does not change; it maintains its gaussianity

# Q & U in a Dual-Beam Experiment

Make a measurement of the sky, $d_i$

Contains T, Q, U components

$$d = T + Q \times \cos 2\alpha + U \times \sin 2\alpha$$

Total signal

$$Q = \frac{1}{2}[d(0^\circ) - d(90^\circ)]$$

$$U = \frac{1}{2}[d(45^\circ) - d(135^\circ)]$$

# Polarization Field

- Define the polarization field (tensor of rank 2) and its Fourier transform (Q and U are the Stokes parameters)

$$Q' + iU' = (Q + iU)e^{2i\phi_x}$$

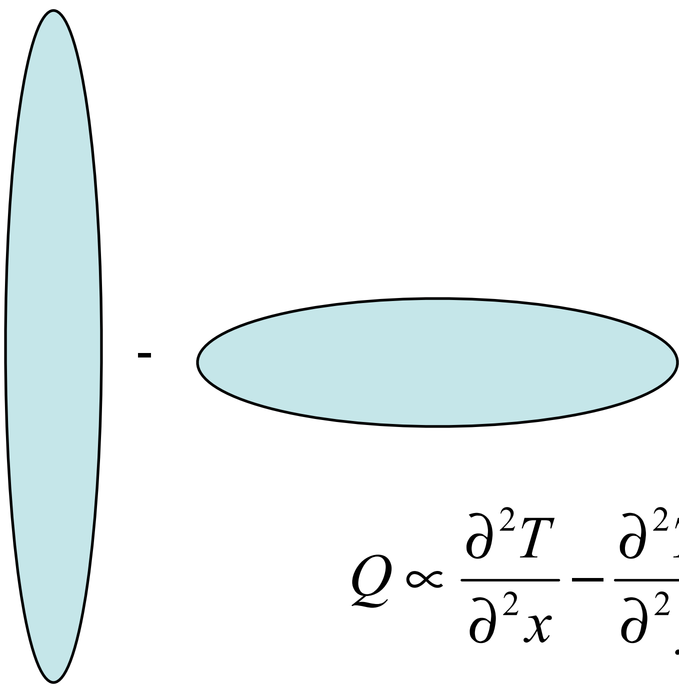
$$Q + iU = \int \frac{d^2\vec{l}}{2\pi}\left[E(\vec{l}) + iB(\vec{l})\right]e^{2i\phi_l}e^{i\vec{l}\cdot\vec{\theta}}$$

- E and B are the Fourier coefficients of the polarization field, they are non-local and the decomposition into E and B is not unique in case of partial sky coverage

# Differential Ellipticity

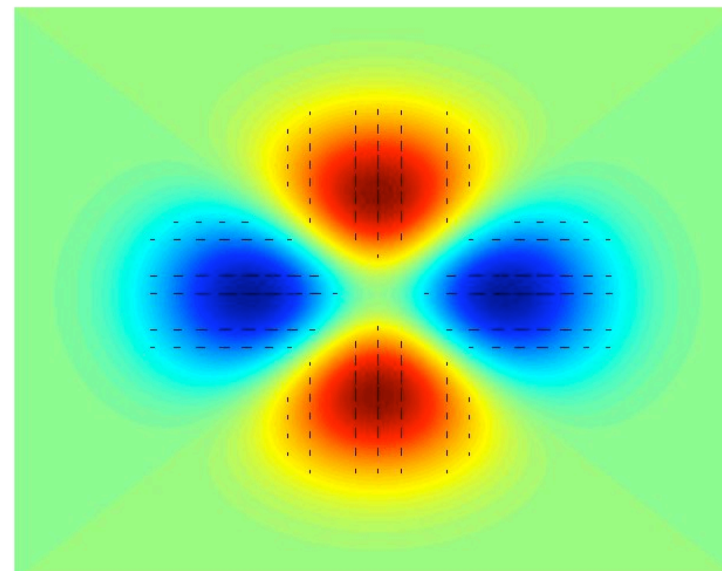
$T_1$ $T_2$

For an unpolarized point source



\- =

Diff. ellipticity



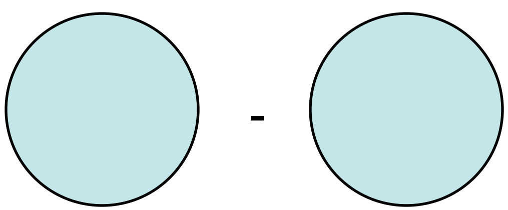
$$Q \propto \frac{\partial^2 T}{\partial^2 x} - \frac{\partial^2 T}{\partial^2 y}$$

$$U \propto 2\frac{\partial^2 T}{\partial x \partial y}$$

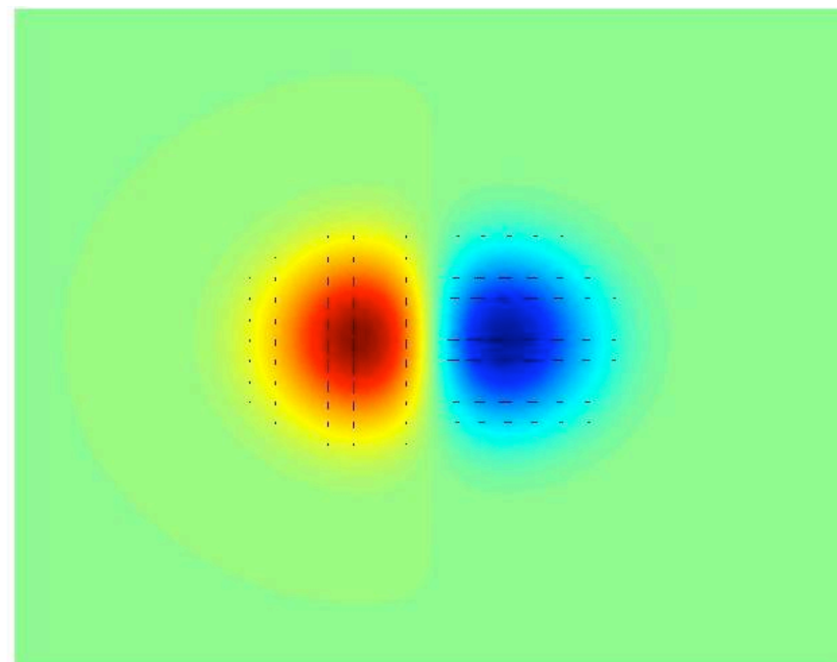
# Differential Pointing

For an unpolarized point source

$T_1$ $T_2$

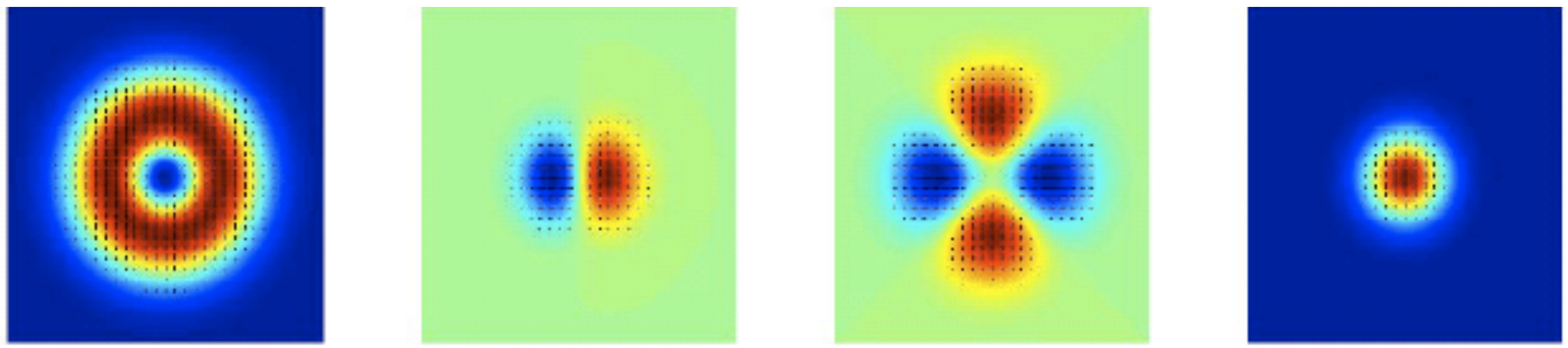


- = Diff. ellipticity



$$\propto \nabla T \oplus \nabla^2 T$$

# Spatial Signature



$$\frac{S}{N} = \int \frac{d^2\vec{l}}{(2\pi)^2} \frac{C_l^B}{C_l^{B,sys}}$$

Haehnelt & Tegmark, 1996

# Map-Domain Pipeline

- Beam systematics are associated with temperature variations over the beam scale

Sky realization ®

first and second derivatives are computed ®

convolving with the beams ®

differencing the maps ®

computing the spurious power spectra

# Frequency-Domain Pipeline

Real space:

$$T(\vec{x}) \circledR\ T \otimes W$$

$$W(\vec{x}) = \frac{1}{2\pi\sigma_x\sigma_y}\exp[-\frac{(x-\rho_x)^2}{2\sigma_x^2} - \frac{(y-\rho_y)^2}{2\sigma_y^2}]$$

Fourier space:

$$\tilde{T}_l \circledR\ \tilde{T}_l \times \tilde{W}_l$$

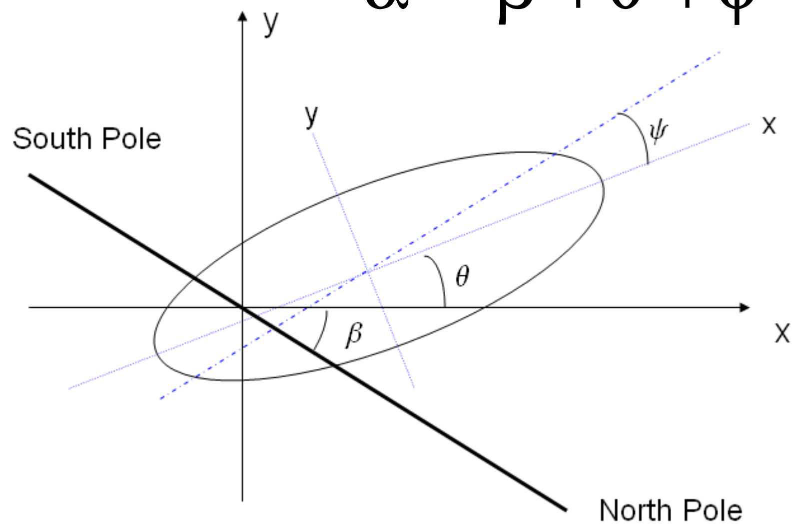
$$e \equiv \frac{\sigma_x - \sigma_y}{\sigma_x + \sigma_y}$$

$$\tilde{W}(\vec{l}) = \exp(-\frac{l_x^2\sigma_x^2}{2} - \frac{l_y^2\sigma_y^2}{2} + i\vec{l} \times \vec{\rho})$$

$$\Rightarrow \tilde{W}(\vec{l}) = \exp\{-\tfrac{(l\sigma)^2}{2} - e(l\sigma)^2\cos 2[(\phi_1 - \chi_1)] + il\rho\cos(\phi_1 - \chi_2)\}$$

$$\tilde{Q}(\vec{l}) = \frac{1}{2}[\tilde{W}_1(\vec{l}) - \tilde{W}_2(\vec{l})] \times \tilde{T}(\vec{l})$$

$$\alpha = \beta + \theta + \psi$$




$$e^{il\rho \cos(\phi_l - \phi_\rho)} = \sum_{n=-\infty}^{\infty} i^n J_n(l\rho) e^{in(\phi_l - \phi_\rho)}$$

$$J_n(iz) = i^n I_n(z)$$

$$\Rightarrow \widetilde{W}(\vec{l}) \equiv \sum_{n,m=-\infty}^{\infty} W_{m,n} e^{i(2m+n)(\phi_l - \alpha)}$$

$$W_{m,n}^{gauss} \equiv e^{-l^2\sigma^2} i^{2m+n} I_m(z) J_n(l\rho) e^{i(2m+n)\psi - in\theta}$$

$$W_{m,n}^{NG} \equiv i^{m+n} J_n(l\rho) e^{i(m+n)\psi + in\vartheta} \int r \times dr J_m(lr) \int d\phi_r W(\vec{r}) e^{-i(m+n)\phi_r}$$

With HWP : $\alpha$ ® $\alpha + 2\varphi\, t$

$$\tilde{Q} = g_1(l,\delta)\tilde{T}(\vec{l})$$

$$\tilde{U} = g_2(l,\delta)\tilde{T}(\vec{l})$$

$$E = \tilde{Q}\cos 2\phi_l + \tilde{U}\sin 2\phi_l$$

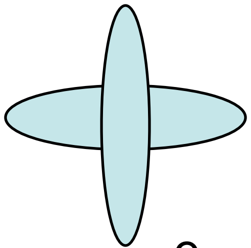
$$B = -\tilde{Q}\sin 2\phi_l + \tilde{U}\cos 2\phi_l$$

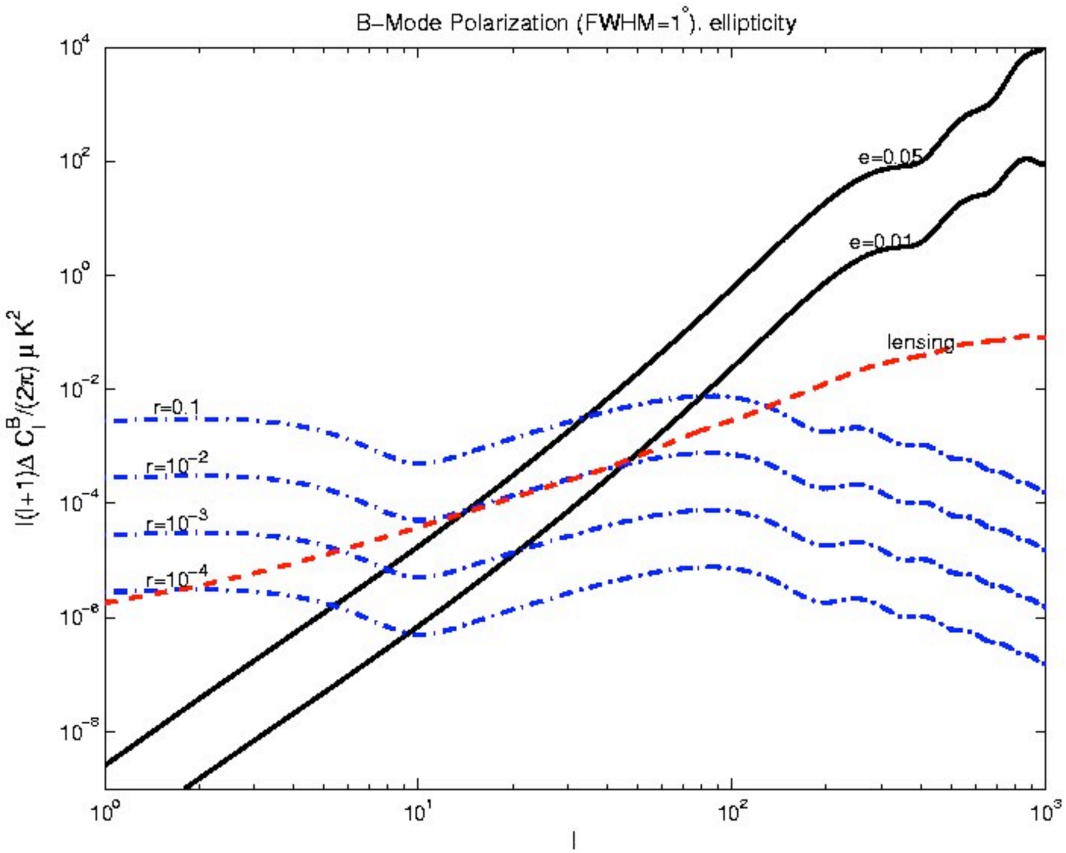
$$C_l^{XY} = \frac{\int X(\vec{l})Y^*(\vec{l})d\phi_l}{2\pi}$$

$$X, Y \in \{T, E, B\}$$

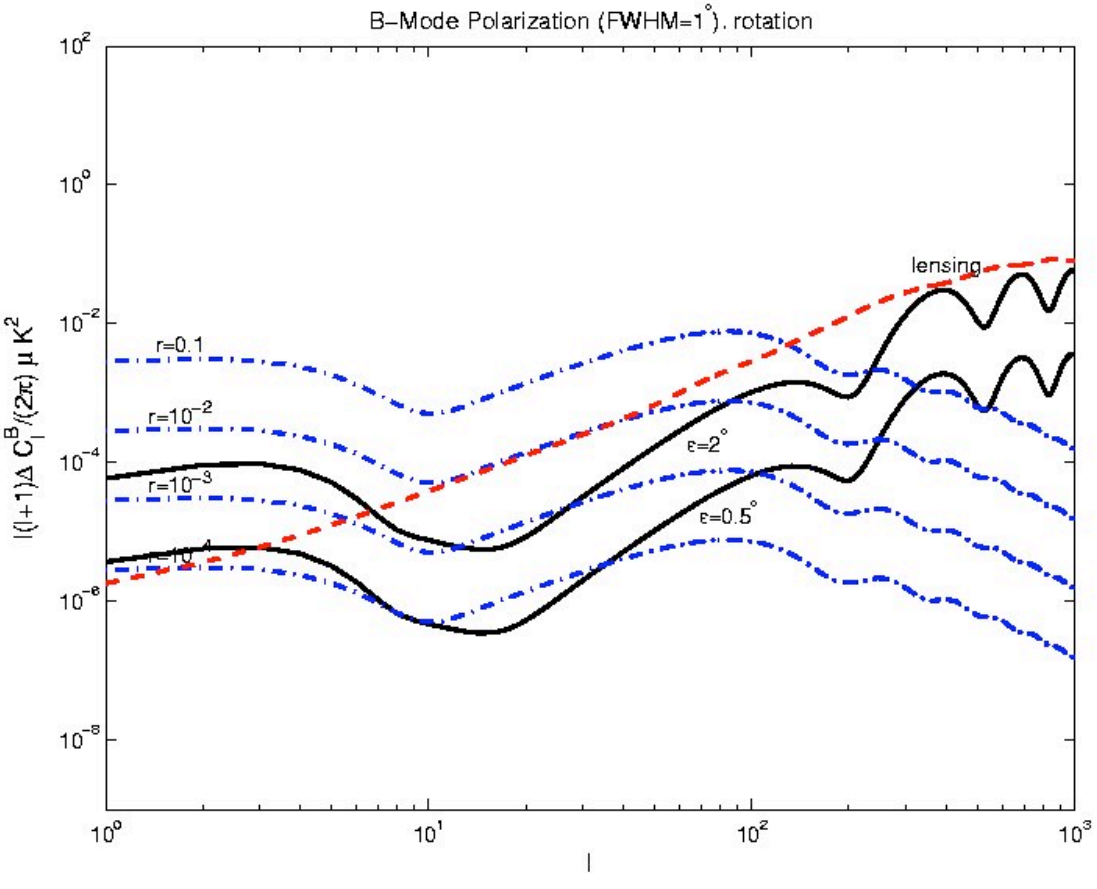
Employ statistical isotropy of underlying sky

# B-Mode Polarization ($1^0$ beam): Diff. Ellipticity



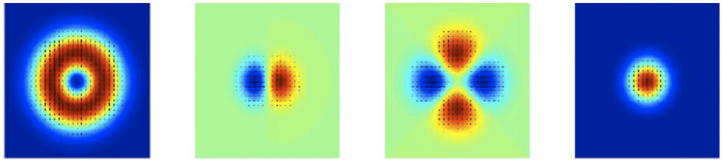
# B-Mode Polarization: beam Rotation



# Spin Classes of Systematics

| Systematic Error | Description | Azimuthal Symmetry | Potential Effect |
|---|---|---|---|
| *Main Beam Effects – Instrumental Polarization* | | | |
| **Δ Beam Size** | $FWHM_E \neq FWHM_H$ | Monopole: spin 0 | $T \rightarrow B$ |
| **Δ Gain** | Mismatched gains, Mismatched coatings | Monopole: spin 0 | $T \rightarrow B$ |
| **Δ Beam Offset** | Pointing E ≠ Pointing H | Dipole: spin 1 | $\nabla T \rightarrow B$ |
| **Δ Ellipticity** | $e_E \neq e_H$ | Quadrupole: spin 2 | $\nabla^2 T \rightarrow B$ |
| **Satellite Pointing** | Q and U beams offset | Complex | $\nabla T \rightarrow B$, $E \rightarrow B$ |
| *Main Beam Effects – Cross Polarization* | | | |
| **Δ Rotation** | E & H not orthogonal | Quadrupole: spin 2 | $E \rightarrow B$ |
| **Pixel Rotation** | $E \perp H$ but rotated w.r.t. beam's major axis | Quadrupole: spin 2 | $E \rightarrow B$ |
| **Optical Cross-Pol** | Birefringence | Quadrupole: spin 2 | $E \rightarrow B$ |

- The various fields couple to moments of non-ideal scanning strategy to generate polarization, i.e. spin +2, -2 fields



# Beam Systematics in the Case of Non-ideal Scanning

$$d(\mathbf{p}) = A(\mathbf{p})m(\mathbf{p}) + n(\mathbf{p})$$

$$m = (T, Q - iU, Q + iU)$$

$$A = (1, \frac{1}{2}e^{2i\alpha}, \frac{1}{2}e^{-2i\alpha})$$

$$\widetilde{m}(\mathbf{p}) = \left( \sum_{j \in \mathbf{p}} A_j^T A_j \right)^{-1} \left( \sum_{j \in \mathbf{p}} A_j^T d_j \right)$$

in case of white noise

- Products of *A* and *T, Q-iU, Q+iU* in real-space correspond to convolutions in Fourier space; non-uniform scanning strategy induces non-gaussianity when coupled to beam and underlying sky

# Fourier-Space Convolutions

$$C_l^{B,gain} = g^2 f_1(\vec{l}) \otimes C_l^T$$

$$C_l^{B,mono} = 4\mu^2 (l\sigma)^4 f_1(\vec{l}) \otimes C_l^T$$

$$C_l^{B,dipole} = f_2(\vec{l}) \otimes [J_1^2(l\rho) C_l^T]$$

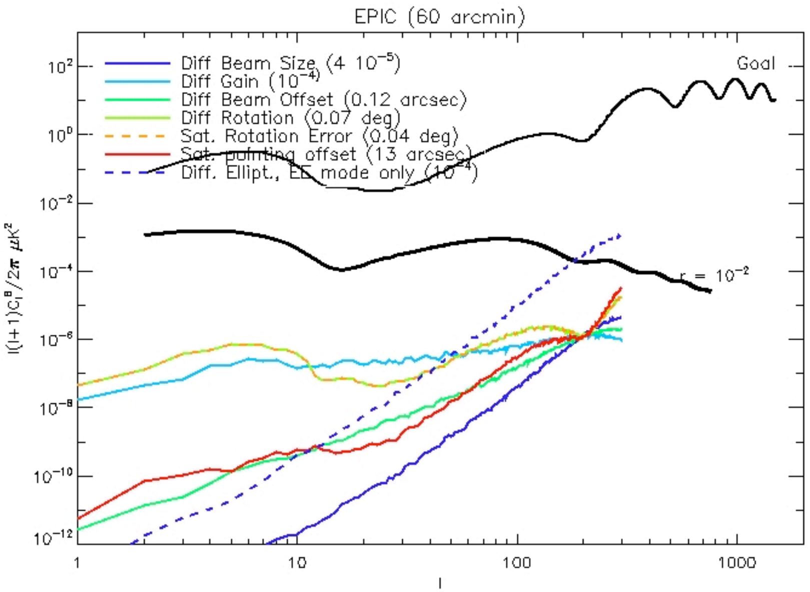
$$\mu \equiv \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2}$$

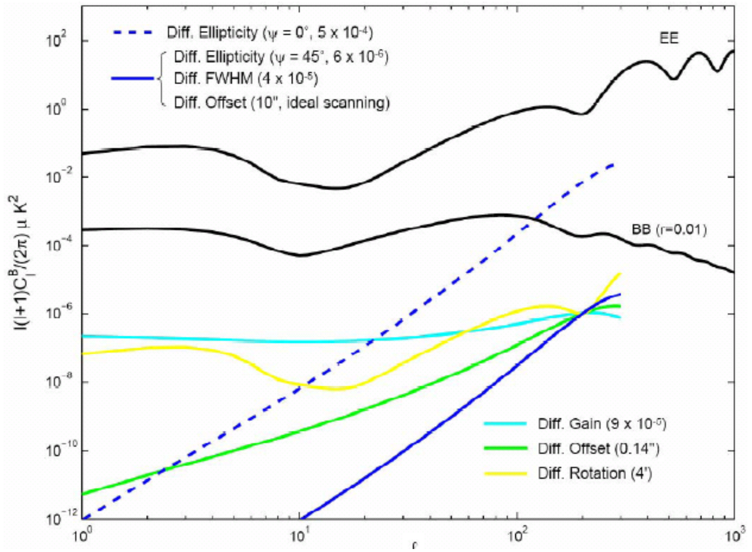
$f_1$ and $f_2$ are functions of the quadrupole and dipole + octupole of the scanning strategy respectively, and vanish in case of ideal scanning

nonvanishing dipole : $\left\langle e^{\pm i\alpha(p)} \right\rangle \neq 0$

nonvanishing quadrupole : $\left\langle e^{\pm 2i\alpha(p)} \right\rangle \neq 0$

# Pipline Comparison

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# Requirements and Goals - Definitions

| Instrument Criteria | Requirements* | Design Goals |
|---|---|---|
| Control systematic errors to negligible levels | Suppress systematic errors to $< 10\%$ of $r = 0.01$ signal, after correction to $\ell \leq 200$, in power | Suppress raw systematic effects to $< 10\%$ of statistical noise level to $\ell \leq 200$, in power |

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- These definitions are motivated by the fact that the main contribution to the error budget is the bias

# Systematic Error Mitigation

| Systematic Error | Goal Suppression | Mitigation | Heritage |
|---|---|---|---|
| *Main Beam Effects – Instrumental Polarization* | | | |
| **Δ Beam Size** | $(\sigma_1-\sigma_2)/\sigma < 4 \times 10^{-5}$ | Half wave plate in front of telescope<br>Refracting telescope<br>Scan crossings | SPIDER & SPUD[‡]<br>N/A<br>BICEP[†] & SPIDER[‡] |
| **Δ Gain** | $(g_1-g_2)/g < 10^{-4}$ | | |
| **Δ Beam Offset** | $\Delta\theta < 0.14''$ raw scan<br>$\Delta\theta < 10''$ symm. scan | | |
| **Δ Ellipticity** | $\Delta e < 5 \times 10^{-4}$, $\psi = 0°$<br>$\Delta e < 6 \times 10^{-6}$, $\psi = 45°$ | | |
| **Satellite Pointing** | $< 12''$ | Dual analyzers | Planck |

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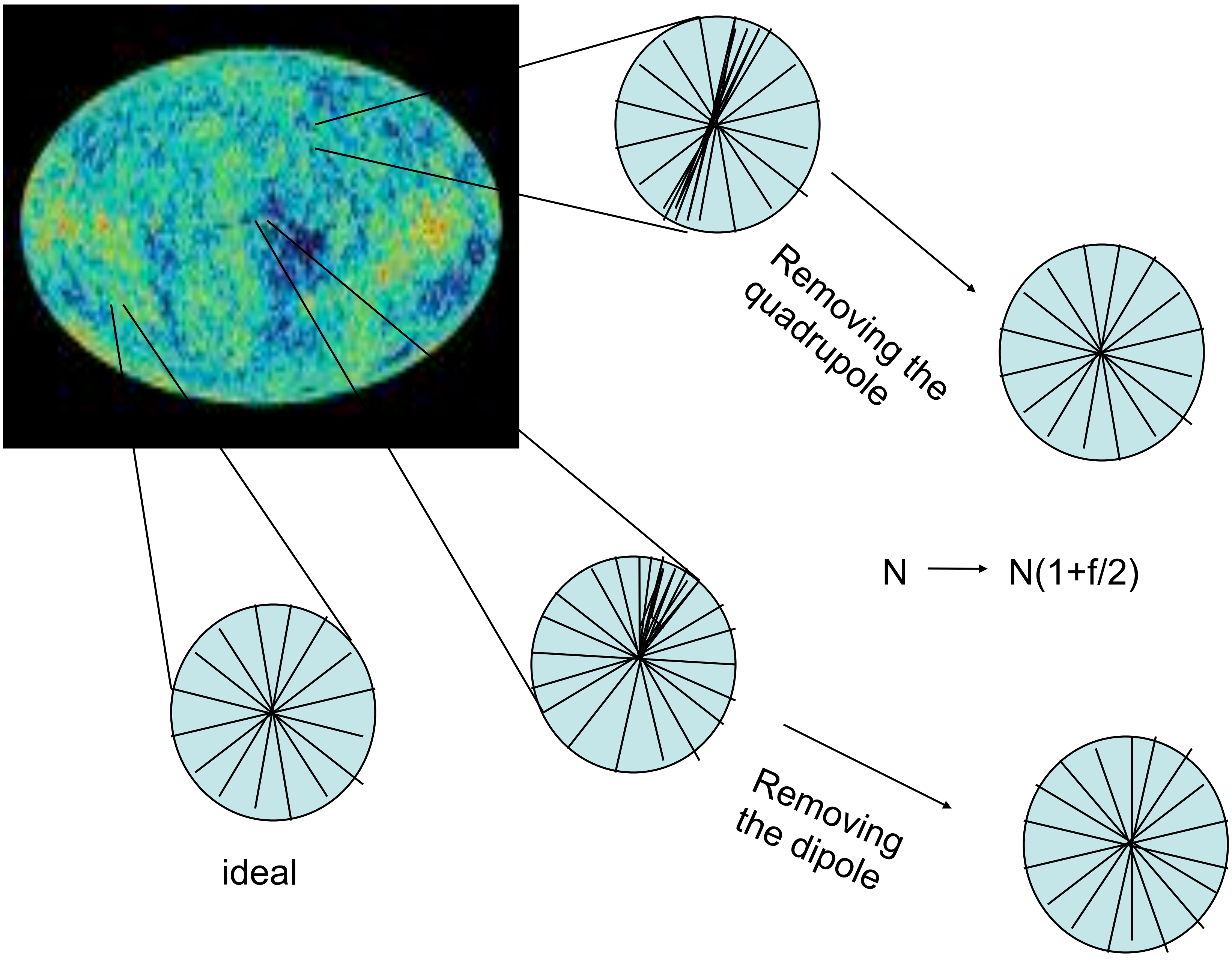
# Post-scanning Idealization

- Differential gain, beamwidth couple to the quadrupole moment of the scanning strategy
- Differential pointing couples to the dipole moment
- Experiments with reasonable scanning can be benefited from throwing away the dipole, quadrupole and octupole multipoles of the data



Removing the quadrupole
N → N(1+f/2)
Removing the dipole
ideal

# Computational Efficacy

- Those systematics that depend on the scanning details can be calculated in two stages: a single computation of the power spectrum of the scanning strategy + multiple calculations of the beam-sky coupling . This should significantly speed-up parameter estimation.

# Parameter Estimation

Comparing observed power spectra $C_l$

with theoretical power spectra $\hat{C}_l = C_l + Ne^{l^2\theta_b^2} + C_l^{sys}$

The lensing - reconstruction noise level
also depends on the observed power spectra
with the increased variance contributed
by temperature - leakage to the B - mode;
this affects mainly those experiments which
benefit from the E - B estimators

# Parameter Inference

$$F_{ij} = \left\langle -\frac{\partial^2 L}{\partial \lambda_i \partial \lambda_i} \right\rangle$$

$$\sigma(\lambda_i) = \sqrt{(F^{-1})_{ii}}$$

$$F_{ij} = \tfrac{1}{2} \sum_l (2l+1) f_{sky} \, \mathrm{Trace}[\mathrm{C}^{-1} \frac{\partial C}{\partial \lambda_i} \mathrm{C}^{-1} \frac{\partial C}{\partial \lambda_j}]$$

$$\hat{C}_l = C_l + N e^{l^2 \theta_b^2} + C_l^{sys}$$

# Non-uniform Scanning, and CMB Lensing

- In this case the local weight of data-points in real-space transform to a non-trivial convolution in multipole-space. This mode-coupling induces non-gaussianity and in particular can be confused with CMB lensing by the LSS (under investigation).

# Summary

- Working in Fourier space from the outset
- Scanning strategy and
  reducible / irreducible systematics
- Full analytic description, including scanning strategy, applicable to any beam shape
- Impact on parameter estimation
- Data "manipulation"
- E-B: monitoring systematics and new physics


Shimon, Keating, Ponthieu and Hivon (2008) PRD 77, 083003

Miller, Shimon and Keating (2008), arXiv:0806.3096

# Acknowledgments

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# Beam and angles