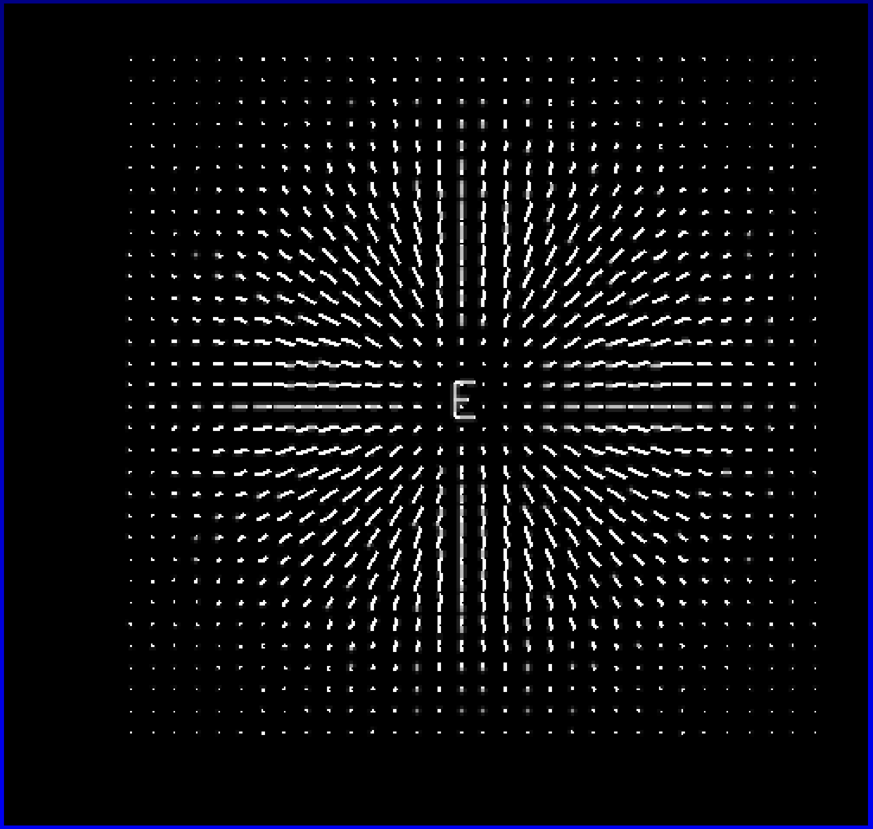
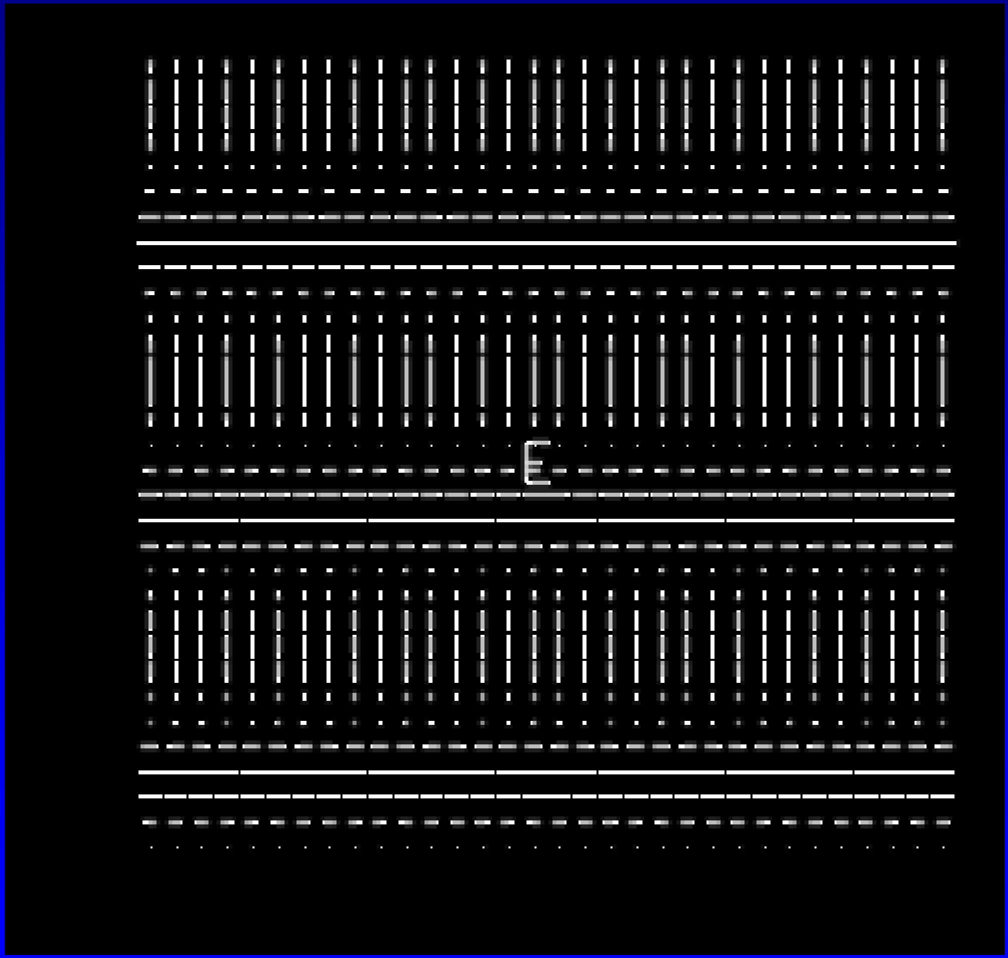
E/B Separation



Ted Bunn

University of Richmond

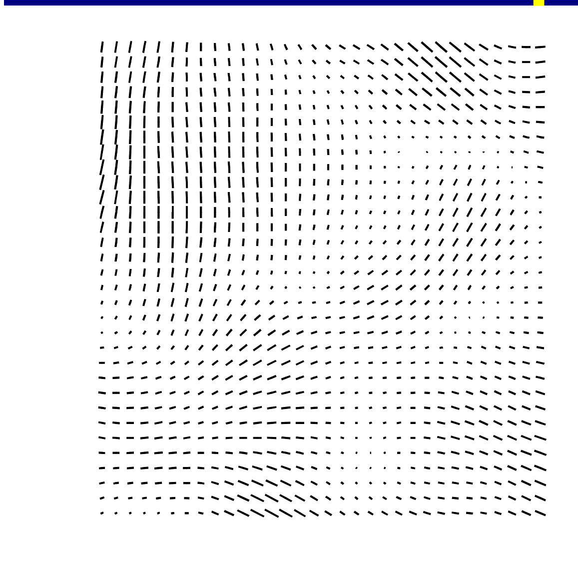
One Fourier mode



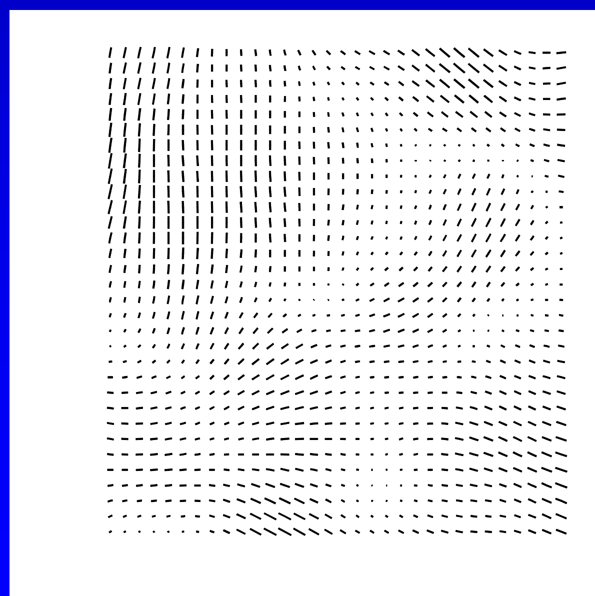
E mode: Polarization parallel / perpendicular to **k**.

B mode: Polarization at 45º to **k**.

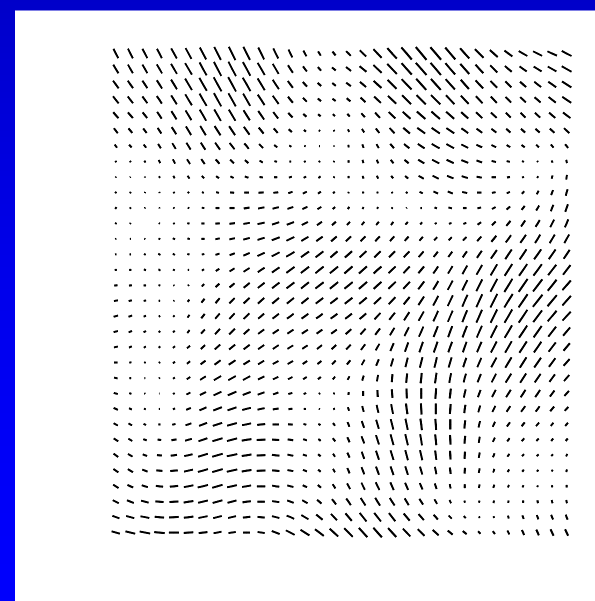
The ideal situation



E



B x 10

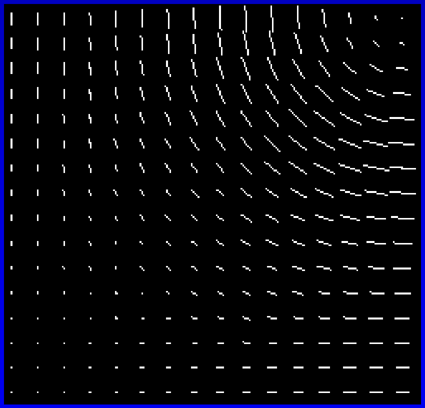


Is E-B Separation Necessary?

- In principle, No!
  - Just calculate $L(C_E, C_B)$ directly from Q, U maps (or even from TOD).
- E, B separation is useful in practice:
  - Exact likelihood calculation is impractical for large data sets.
  - Diagnosis of foregrounds, systematic errors.
  - Rhetoric (convincing the world we know what we've seen).
- Intuition about EB separation issues is useful in experiment design.
  - Optimizing sky coverage, pixelization, etc.

Ambiguity

- *E*-*B* decomposition is unique *given full sky coverage.*
- In a region with a boundary, you can fool yourself:



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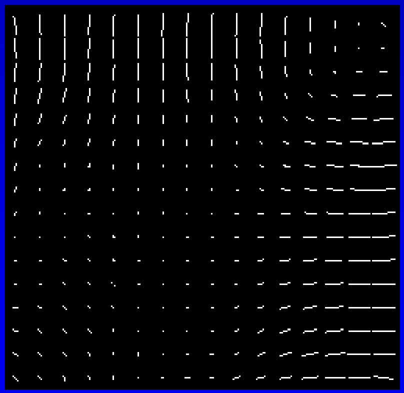

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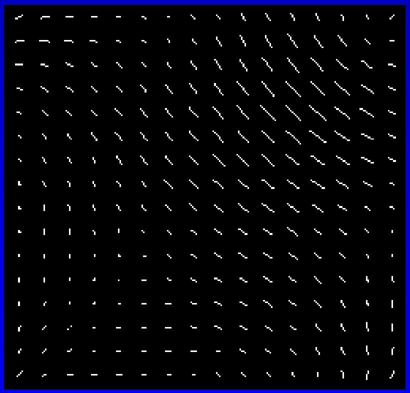


"Really" an E mode

Naïve Fourier-space E/B decomposition

The Simple Picture

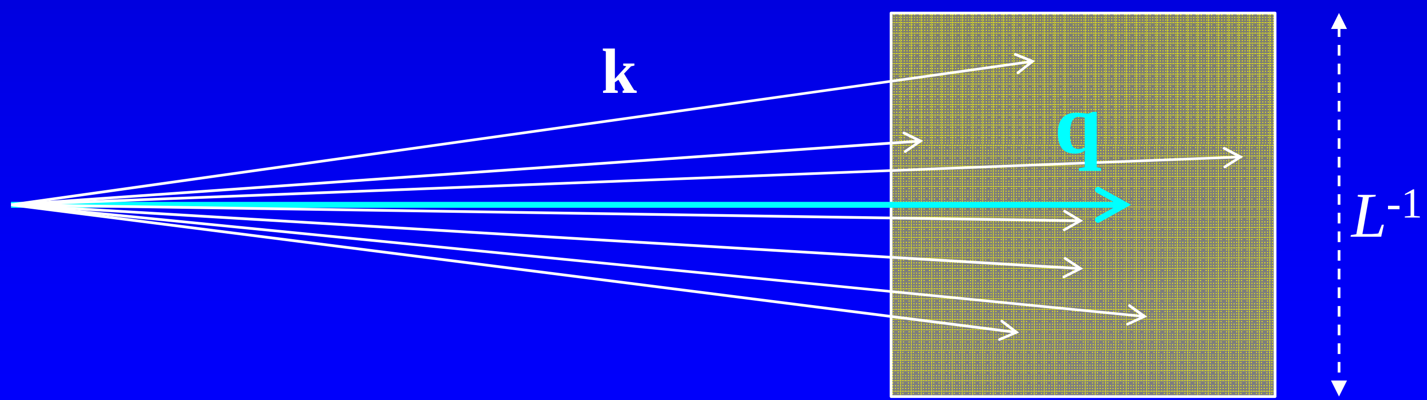
Experiment covers a small patch of sky of area L^2

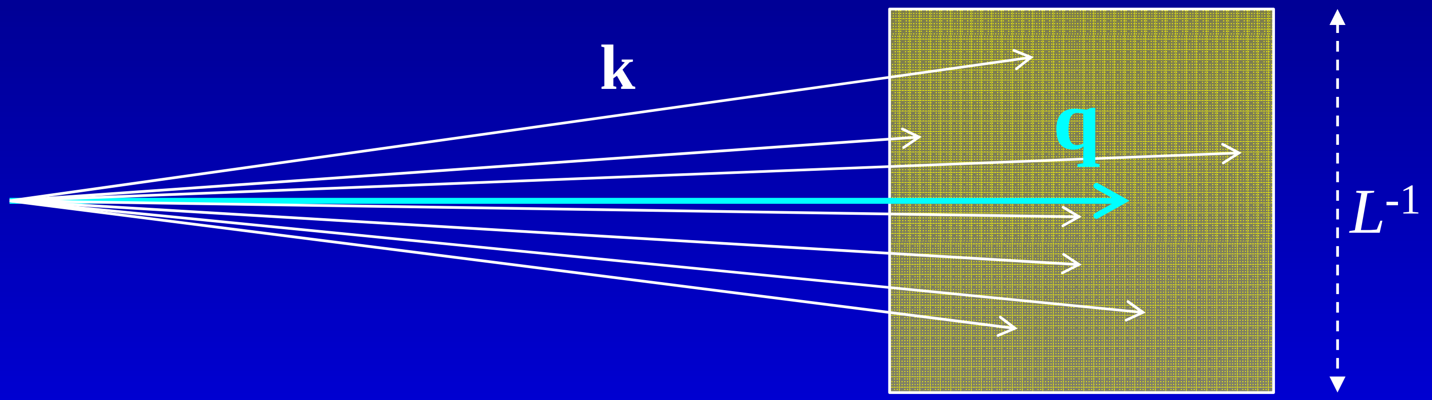
Estimated Fourier coefficients: $\tilde{\mathbf{a}}(\mathbf{q}) = \frac{1}{L^2}\int_{L^2} d^2x\, \mathbf{p}(\mathbf{x}) e^{-i\mathbf{q}\cdot\mathbf{x}}$

Each estimated mode is a mixture of true Fourier modes:

$$|\,\mathbf{k} - \mathbf{q}\,| \sim 1/L.$$

Mixing of different directions in k space ➔ E/B mixing.





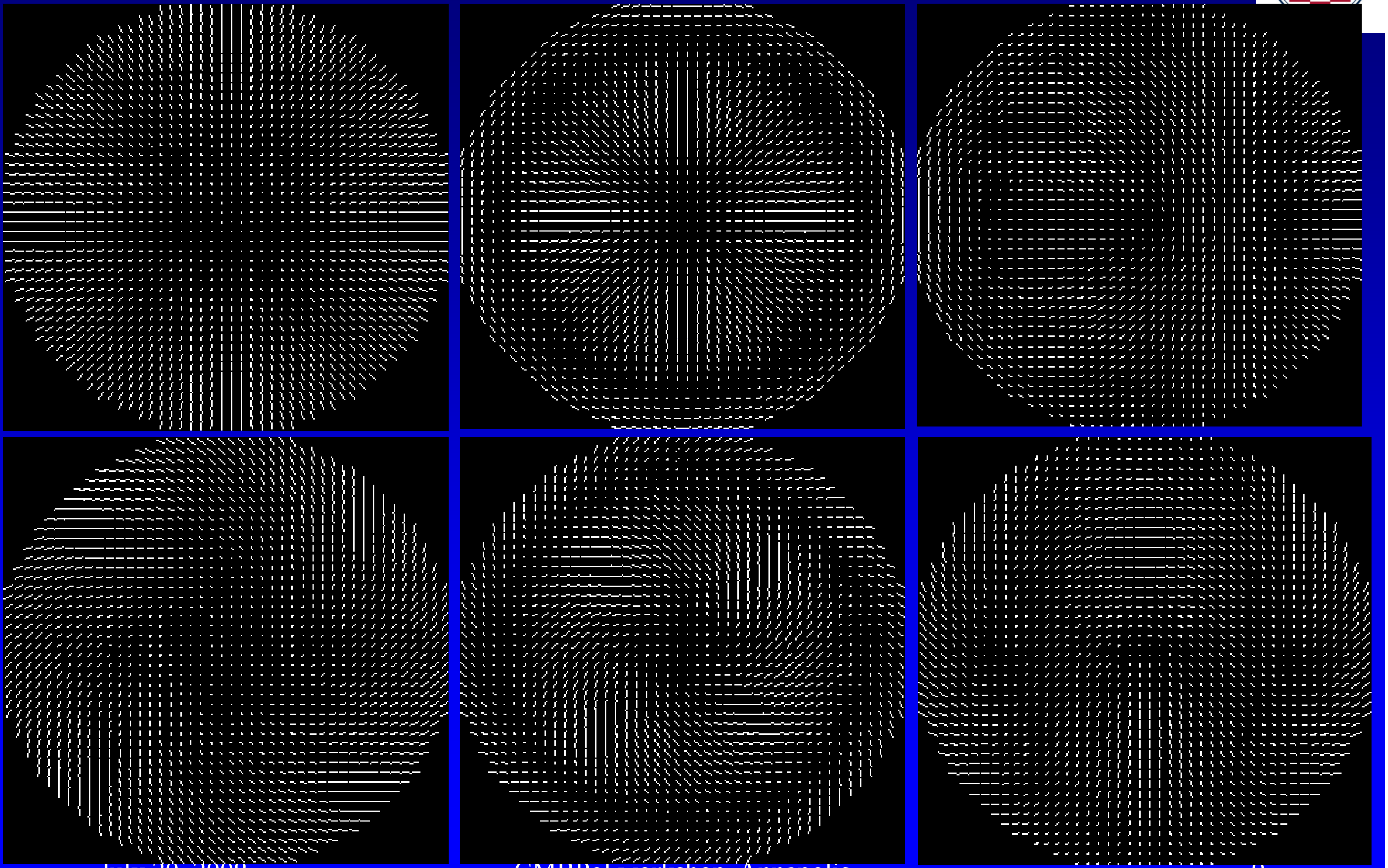
- Measurement of $U(\mathbf{q})$ “should” be B-component.
- Actually contains E-component of order $(qL)^{-1}$
- Since E >> B, some nominal B-modes are actually E-dominated.
- Spread in angles is smaller at higher wavenumber
  ➔ E/B mixing is worst at largest scales observed

Pure and ambiguous modes

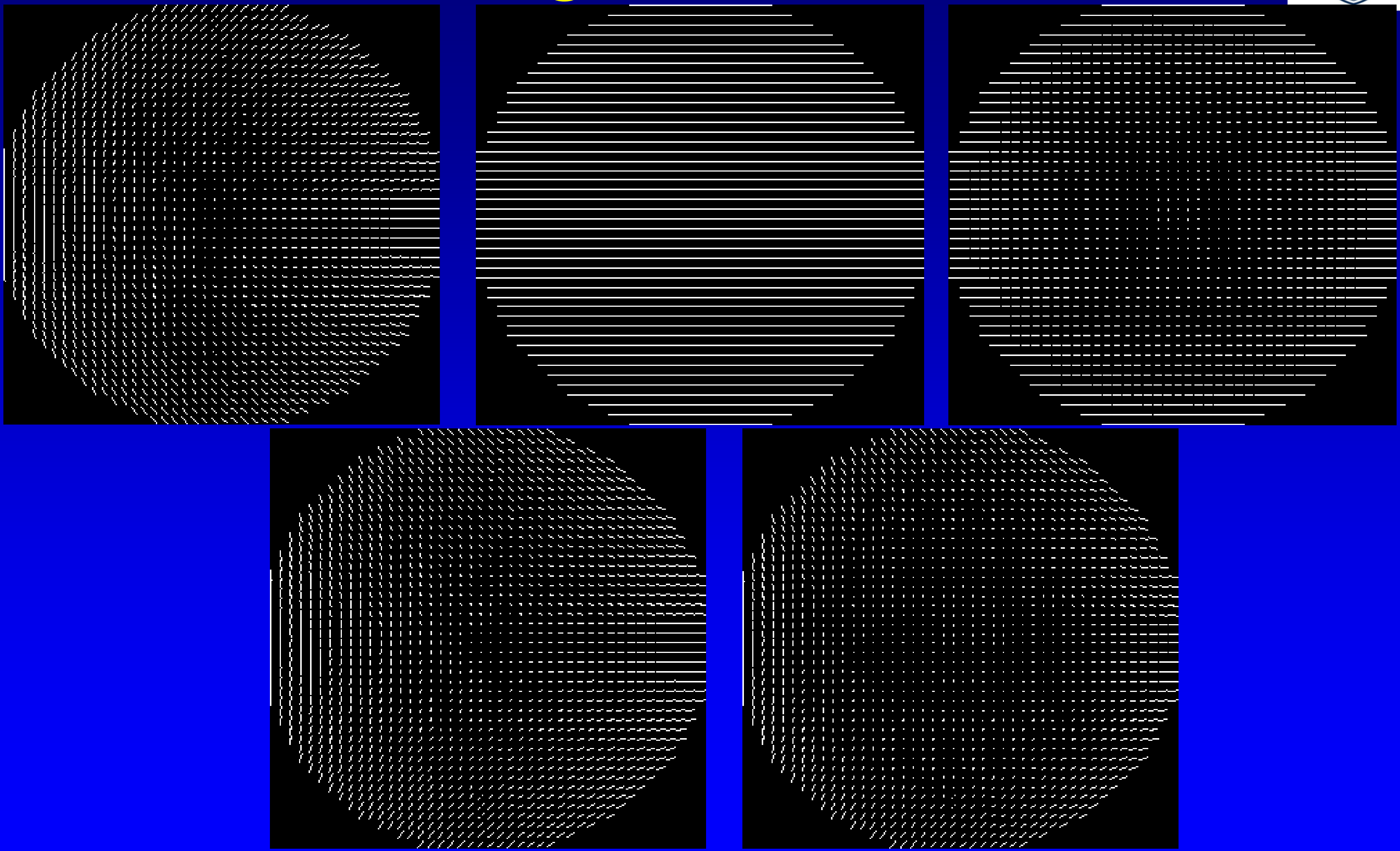
- Analogy: curl-free and divergence free vector fields.
- Over a finite patch, there are ambiguous fields that are both curl-free and divergence-free.
- If you project out those modes, remaining component is uniquely decomposable into “pure” modes.
- For a reasonably “nice-shaped” region of length scale L, ambiguous modes probe wavenumbers $\sim n/L$, with two modes per n.
- $N_{\mathrm{amb}} / N_{\mathrm{pure}} \sim 1$ on largest scales probed, but drops as 1/wavenumber.
- SNR is always highest for largest-scale modes, though, so this hurts more than you might think.

Bunn et al 2003
Lewis et al 2003

Pure E and B modes



Ambiguous modes

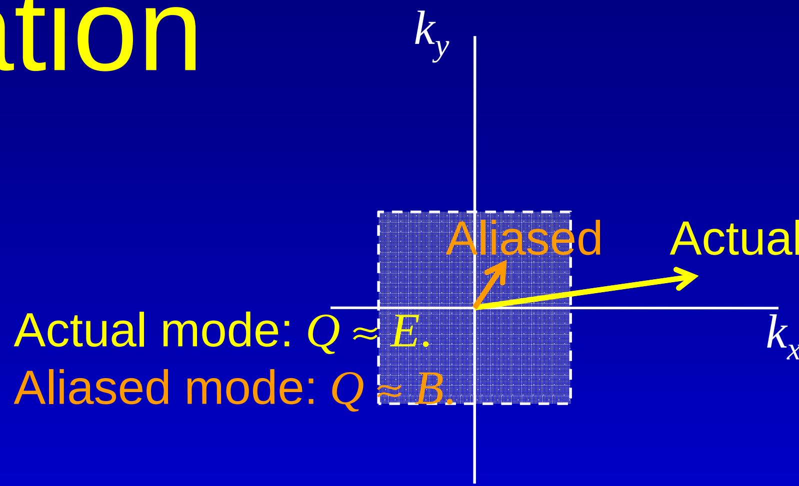


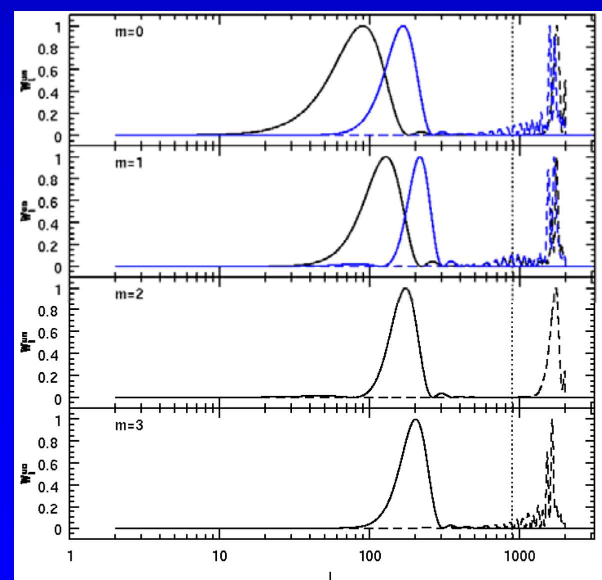
Pixelization

- Aliasing of Fourier modes above the Nyquist frequency completely scrambles the direction of **k**.
- Need to oversample the beam, to make sure Nyquist frequency is very high.

$$e^{-\sigma_b^2 k_{\mathrm{Ny}}^2} < C_k^B / C_{k_{\mathrm{Ny}}}^E$$

- Typically means pixel size ~3 times smaller than beam FWHM.



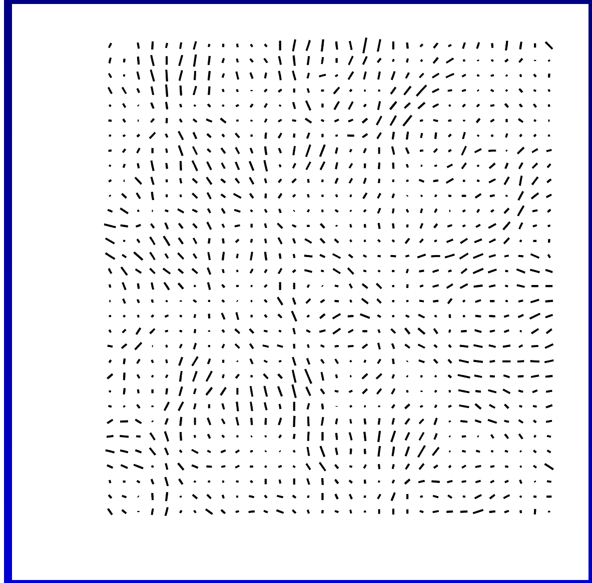


Bunn et al 2003

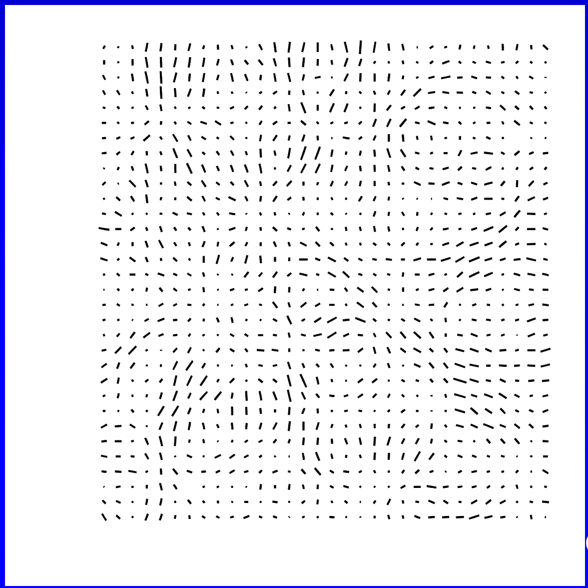
Efficient pure/ambiguous decomposition

- Finding a basis of pure and ambiguous modes is annoying / expensive.
- Much easier to do the decomposition in real space:
  1. Do naïve E/B separation any way you like (anafast / synfast).
  2. “Purify” the B component by finding a curl-free function with given boundary conditions and subtracting it off.
     - (Curl-free piece can be found by relaxation methods, and it’s smooth on the pixel scale.)

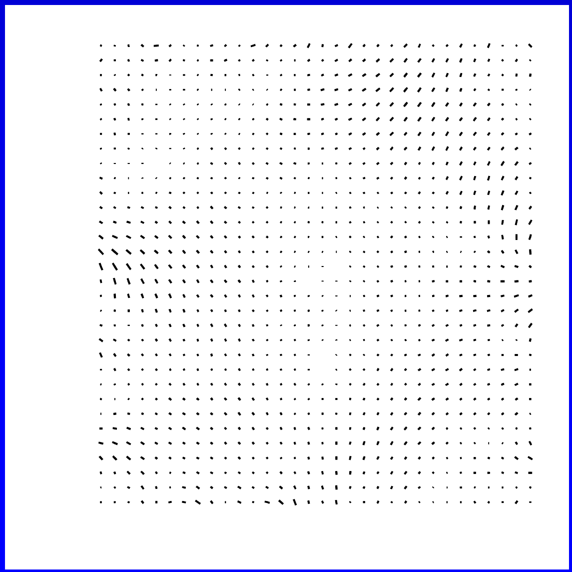
Example



E

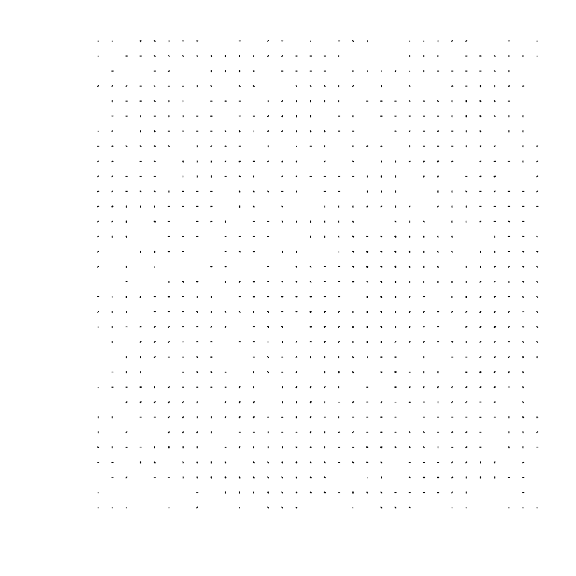


B

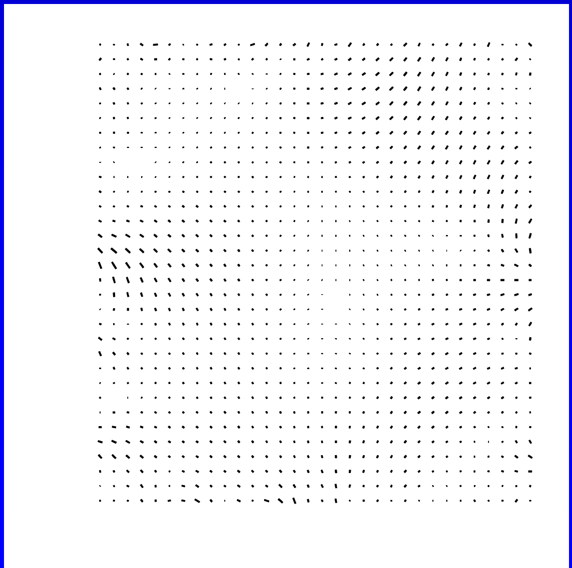


Example

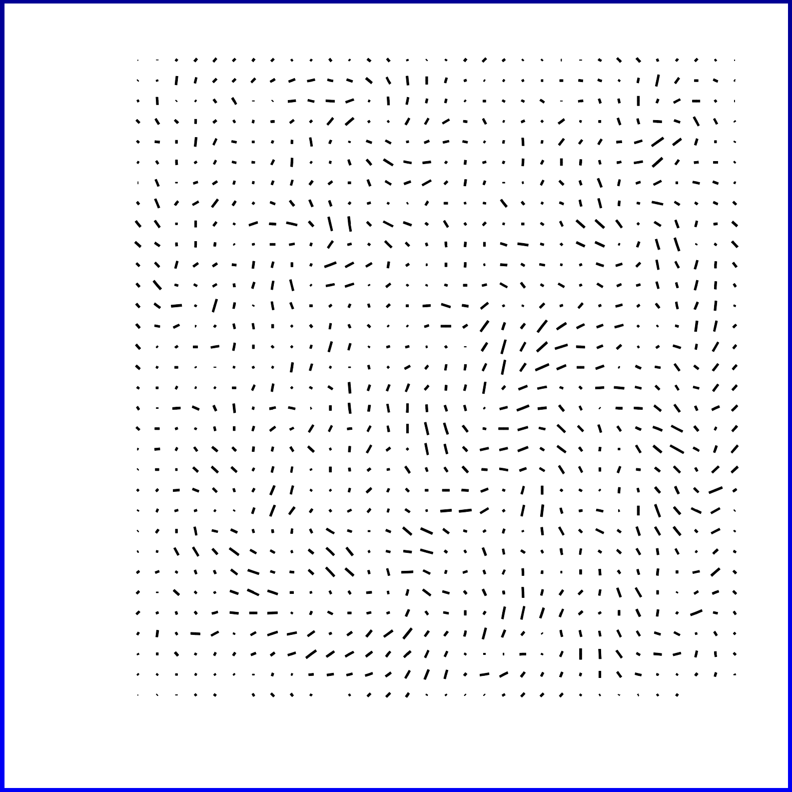
Pure



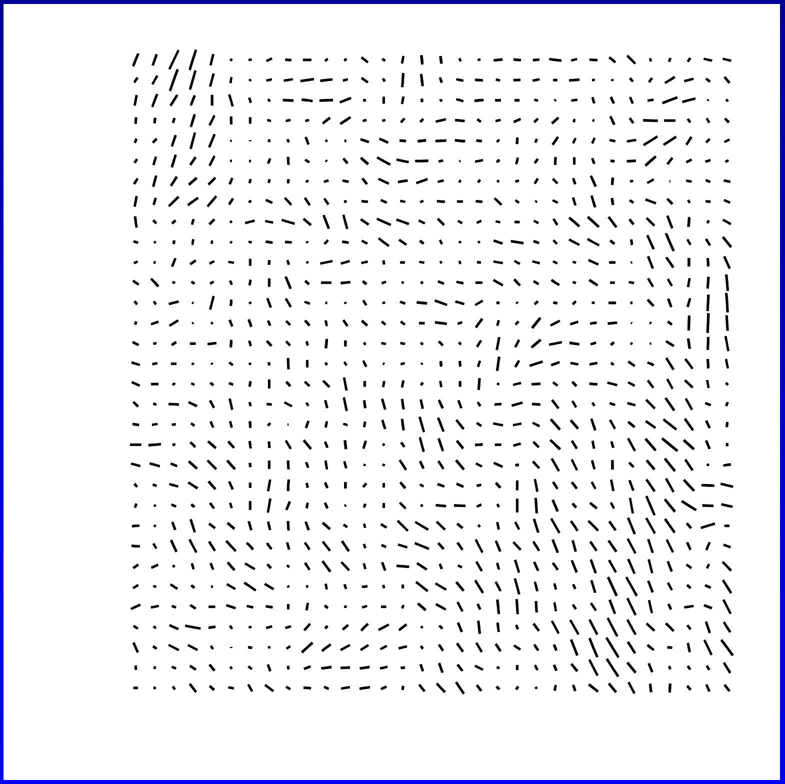
Ambiguous



Example



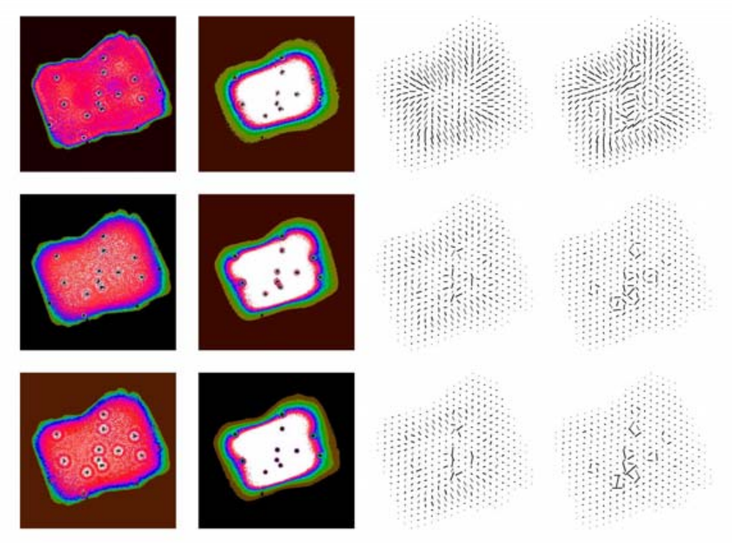
Pure B



Actual B

Pseudo-C_l method

- Smith & Zaldarriaga (2006):
  - Take the “curl” of data to project down onto the pure B subspace.
  - Define pseudo-C_l’s on this space.
  - Other methods besides pseudo-C_l’s might also benefit from working with the curl.
- Method involves derivatives of noisy pixelized data. ➔ Careful apodization of weight functions is crucial to avoid aliasing.



Speculation uncontaminated by calculation: Projecting out ambiguous modes (instead of taking curls) might make implementation of this method easier (or more transparent).

Interferometry

- Lore: Interferometric visibilities live in Fourier space, so they're better for E/B separation.
  - This is too vague to be either true or false!
- Some things that are true:
  - A single visibility pair (V_Q, V_U) contains some E,B information (unlike a single pixel in a map).
  - For small separation between antennas, there's lots of mixing ➔ not much B information.

Interferometry

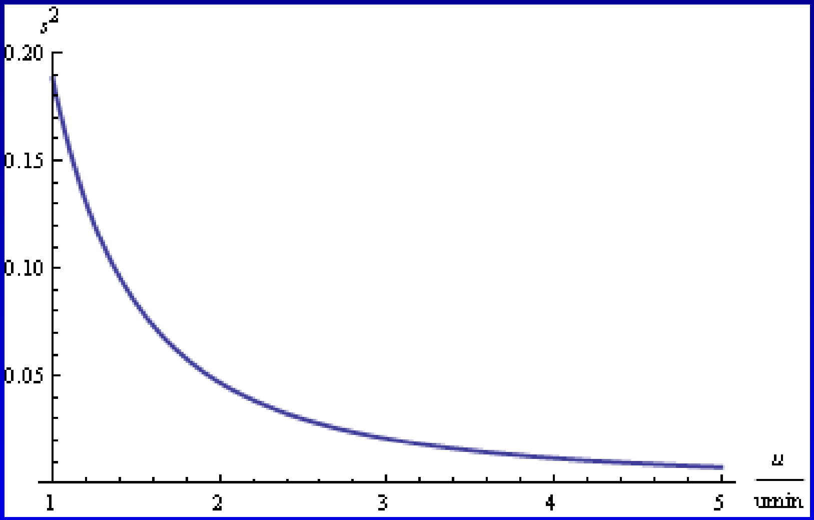
$$\langle |V_Q|^2 \rangle = C_E(1-\overline{s^2}) + C_B\overline{s^2}$$

$$\langle |V_U|^2 \rangle = C_E\overline{s^2} + C_B(1-\overline{s^2})$$

Both visibilities dominated by E unless

$$\overline{s^2} \sim (C_B/C_E) \sim 10^{-2}$$

E/B separation in a single baseline requires antenna separation at least ~ 4 x diameter.



E/B separation for shorter baselines (larger angular scales) requires multiple visibilities / mosaicking to sharpen Fourier-space resolution. Dense sampling of the visibility plane is the key.

Comments

- E-B mixing due to boundary affects primarily largest observed scales.
- Number of ambiguous modes scales with boundary length; rounder is better.
- Aliasing strongly mixes E, B. Oversample!
- Maps can be “purified” in real space without figuring out an entire basis of modes – possibly useful for pseudo-C_l method.
- Interferometric data give good E/B separation in individual visibilities only for long baselines. For shorter baselines, dense sampling in visibility plane and/or mosaicking are needed.