One Fourier mode

E mode: Polarization parallel / perpendicular to $k$.

B mode: Polarization at 45° to $k$.  

July 30, 2008  CMBPol workshop, Annapolis
The ideal situation
Is E-B Separation Necessary?

• In principle, No!
  – Just calculate $L(C_E, C_B)$ directly from $Q, U$ maps (or even from TOD).

• $E, B$ separation is useful in practice:
  – Exact likelihood calculation is impractical for large data sets.
  – Diagnosis of foregrounds, systematic errors.
  – Rhetoric (convincing the world we know what we’ve seen).

• Intuition about $EB$ separation issues is useful in experiment design.
  – Optimizing sky coverage, pixelization, etc.
Ambiguity

- $E$-$B$ decomposition is unique *given full sky coverage*.
- In a region with a boundary, you can fool yourself:

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Really" an E mode = Naïve Fourier-space E/B decomposition +
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The Simple Picture

Experiment covers a small patch of sky of area $L^2$

Estimated Fourier coefficients: $\tilde{a}(q) = \frac{1}{L^2} \int_{L^2} d^2x \, p(x) e^{-i q \cdot x}$

Each estimated mode is a mixture of true Fourier modes:

$|k - q| \sim 1/L$.

Mixing of different directions in $k$ space $\Rightarrow$ E/B mixing.
• Measurement of $U(q)$ “should” be B-component.
• Actually contains E-component of order $(qL)^{-1}$
• Since $E \gg B$, some nominal B-modes are actually E-dominated.
• Spread in angles is smaller at higher wavenumber
  ➔ E/B mixing is worst at largest scales observed
Pure and ambiguous modes

- Analogy: curl-free and divergence free vector fields.
- Over a finite patch, there are ambiguous fields that are both curl-free and divergence-free.
- If you project out those modes, remaining component is uniquely decomposable into “pure” modes.
- For a reasonably “nice-shaped” region of length scale $L$, ambiguous modes probe wavenumbers $\sim n/L$, with two modes per $n$.
- $N_{\text{amb}} / N_{\text{pure}} \sim 1$ on largest scales probed, but drops as $1/$wavenumber.
- SNR is always highest for largest-scale modes, though, so this hurts more than you might think.

Bunn et al 2003
Lewis et al 2003
Pure E and B modes
Ambiguous modes
Pixelization

- Aliasing of Fourier modes above the Nyquist frequency completely scrambles the direction of $k$.
- Need to oversample the beam, to make sure Nyquist frequency is very high.

$$e^{-\sigma_b^2 k^2_{Ny}} < \frac{C_B}{C_E}$$

- Typically means pixel size $\sim 3$ times smaller than beam FWHM.

Bunn et al 2003
Efficient pure/ambiguous decomposition

• Finding a basis of pure and ambiguous modes is annoying / expensive.
• Much easier to do the decomposition in real space:
  1. Do naïve E/B separation any way you like (anafast / synfast).
  2. “Purify” the B component by finding a curl-free function with given boundary conditions and subtracting it off.
     • (Curl-free piece can be found by relaxation methods, and it’s smooth on the pixel scale.)
Example

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Example

Pure

Ambiguous

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Example

Pure B

Actual B

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Pseudo-$C_l$ method

- Smith & Zaldarriaga (2006):
  - Take the “curl” of data to project down onto the pure B subspace.
  - Define pseudo-$C_l$’s on this space.
  - Other methods besides pseudo-$C_l$’s might also benefit from working with the curl.

- Method involves derivatives of noisy pixelized data. Careful apodization of weight functions is crucial to avoid aliasing.

Speculation uncontaminated by calculation: Projecting out ambiguous modes (instead of taking curls) might make implementation of this method easier (or more transparent).
Interferometry

• Lore: Interferometric visibilities live in Fourier space, so they’re better for E/B separation.
  – This is too vague to be either true or false!

• Some things that are true:
  – A single visibility pair \((V_Q, V_U)\) contains some E,B information (unlike a single pixel in a map).
  – For small separation between antennas, there’s lots of mixing ➔ not much B information.
Interferometry

\[ \langle |V_Q|^2 \rangle = C_E (1 - \overline{s^2}) + C_B \overline{s^2} \]

\[ \langle |V_U|^2 \rangle = C_E \overline{s^2} + C_B (1 - \overline{s^2}) \]

Both visibilities dominated by \( E \) unless

\[ \overline{s^2} \sim \left( \frac{C_B}{C_E} \right) \sim 10^{-2} \]

E/B separation in a single baseline requires antenna separation at least \( \sim 4 \times \) diameter.

E/B separation for shorter baselines (larger angular scales) requires multiple visibilities / mosaicking to sharpen Fourier-space resolution. Dense sampling of the visibility plane is the key.
Comments

• E-B mixing due to boundary affects primarily largest observed scales.
• Number of ambiguous modes scales with boundary length; rounder is better.
• Aliasing strongly mixes E, B. Oversample!
• Maps can be “purified” in real space without figuring out an entire basis of modes – possibly useful for pseudo-$C_l$ method.
• Interferometric data give good E/B separation in individual visibilities only for long baselines. For shorter baselines, dense sampling in visibility plane and/or mosaicking are needed.