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Analytic Approach to CMB Beam Systematics: Effect on Inflation Science, Lensing Extraction and Cosmic Birefringence

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ABSTRACT

The CMB's B-mode polarization provides a handle on several cosmological parameters most notably the tensor-to-scalar ratio, r, and is sensitive to parameters which govern the growth of large scale structure (LSS) and evolution of the gravitational potential. These imprint CMB temperature anisotropy and cause E-to-B-mode polarization conversion via gravitational lensing. The CMB has also the potential to constrain departures from standard electromagnetism and gravitation - for which the 'forbidden' TB and EB correlations are unique 'smoking guns'. However, both primordial gravitational-wave and secondary lensing-induced B-mode signals, as well as the upper limits on TB and EB are very weak and therefore prone to various foregrounds and systematics. In this summary we concisely describe an analytic approach to beam systematics which allows a fast and accurate determination of various beam systematics in certain idealized cases - this tool is particularly useful in the design-phase of CMB experiments like CMBPOL when forecasts of the expected science-return are being made. We also discuss the issue of scanning strategy as a mitigation tool and propagate beam systematics to parameter estimation to determine tolerances on the allowed beam parameters.

1 INTRODUCTION

Beam systematics have been discussed extensively (e.g. Hu, Hedman & Zaldarriaga 2003, Rosset et al. 2004, O'Dea, Challinor & Johnson 2007, Shimon et al. 2008 as well as in the context of specific CMB experiments). All the effects are associated with beam imperfections or beam mismatch in dual beam experiments, i.e. where the polarization is obtained by differencing two signals which are measured simultaneously by two beams with two (ideally) orthogonal polarization axes. Fortunately, several of these effects (e.g. differential gain, differential beamwidth and the first order pointing error - 'dipole'; Hu, Hedman & Zaldarriaga 2003, O'Dea, Challinor & Johnson 2007, Shimon et al. 2008) are reducible with an ideal scanning strategy and otherwise can be cleaned from the dataset by virtue of their non-quadrupolar nature which distinguishes them from genuine CMB polarization signals. Other spurious polarization signals, such as those due to differential ellipticity of the beam, second order pointing errors and beam rotation, persist even in the case of ideal scanning strategy and perfectly mimic CMB polarization. These represent the minimal spurious B-mode signal, residuals which will plague every polarization experiment. We refer to them as 'irreducible beam systematics'. We present a non-perturbative analytic calculation of a few such signals (Shimon et al. 2008). To set tolerance levels on beam imperfections one would like to propagate the resulting systematics through to parameter estimation to assess their impact on cosmological parameters. Our viewpoint is that CMBPOL's primary goal is the detection of the primordial B-mode polarization induced by the stochastic gravitational wave background generated during the inflationary phase (Baumann et al. 2008). Perhaps a comparably major goal of CMBPOL will be the detection of the secondary, lensing-induced, B-mode signal (Smith et al. 2008). While the detection of a non-vanishing cosmic birefringence (CB) will have a far-reaching impact on fundamental physics, its current status among the list of scientific goals of CMBPOL is only secondary. As we discuss below, optimizing beam systematics for the inflation science guarantees that the lensing science

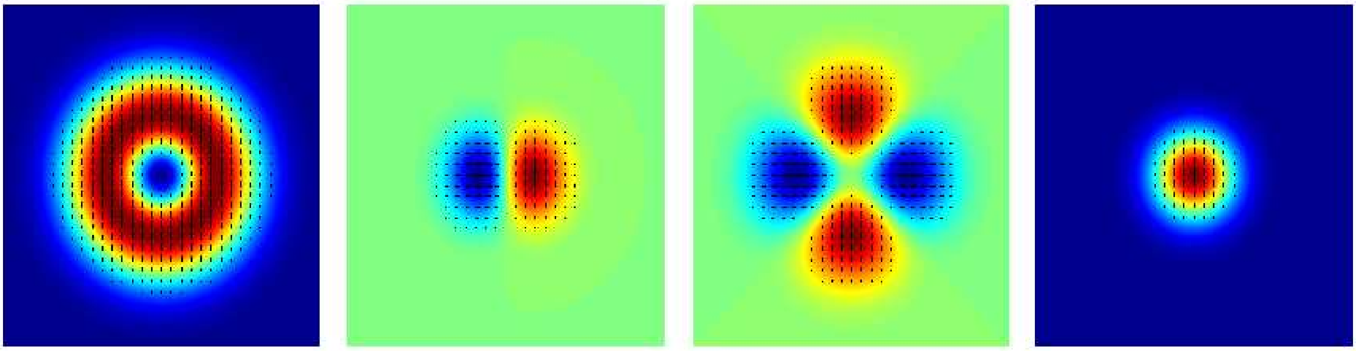

Figure 1. An illustration of the monopole, dipole, quadrupole and gain effects; Q parameter only is depicted.

will be virtually systematics-free. However, it is not so for CB since the smoking guns for this effect are correlations *linear* in B-mode polarization, namely TB and EB, whereas the inflationary and lensing sciences mainly benefit from correlations which are *quadratic* in B. To calculate the effect of beam systematics on parameter uncertainty and bias the Fisher information-matrix formalism is the method of choice although we also experimented with Monte Carlo Markov chain (MCMC)-based parameter estimation in the presence of beam systematics [Miller, Shimon & Keating 2009a (for both inflation and lensing)] and found, as expected, that for bias levels much smaller than the statistical error (uncertainty) the Fisher-matrix-based estimation gives sufficiently accurate estimates. This analysis consists of propagating the systematic errors through to cosmological parameter estimation and assessing its effect on the inferred parameters, especially their induced bias. In the analysis the extra noise due to beam systematics is represented by analytic approximations (Shimon et al. 2008) and includes lensing extraction (Miller, Shimon & Keating 2009a) in the parameter inference process, following Kaplinghat, Knox & Song (2003) and Lesgourgues et al. (2006) for neutrino mass inference from CMB data (and other cosmological parameters).

2 BEAM SYSTEMATICS

Beam systematics due to optical imperfections depend on both the underlying sky, the properties of the polarimeter and on the scanning strategy. A constructive example is the effect of differential pointing. This effect depends on the CMB temperature gradient to first order. The rms CMB temperature gradients at the 1°, $30'$, $10'$, $5'$ and $1'$ scales are ≈ 1.4, 1.5, 3.5, 2.5 and 0.2 μK/arcmin, respectively. Therefore, any temperature difference measured with a dual-beam experiment (with typical beamwidth few arcminutes) with a $\approx 1'$ pointing error and non-ideal scanning strategy (which is dominated by its dipole and octupole moments (Shimon et al. 2008)) will result in a $\approx 1\mu$K systematic polarization which has the potential to contaminate the B-mode signal. Similarly, the systematic induced by differential ellipticity results from the variation of the underlying temperature anisotropy along the two polarization-sensitive directions which, in general, differ in scale depending on the mean beamwidth, degree of ellipticity and the tilt of the polarization-sensitive direction with respect to the ellipse's principal axes. For example, the temperature difference measured along the major and minor axes of a 1° beam with a 2% ellipticity scales as certain components of the second gradient of the underlying temperature (e.g. $Q \propto \frac{\partial^2 T}{\partial x^2} - \frac{\partial^2 T}{\partial y^2}$ and $U \propto 2\frac{\partial^2 T}{\partial x \partial y}$) which on this scale is $\approx 0.2\mu K/\mathrm{arcmin}^2$ and the associated induced polarization is therefore expected to be on the $\approx \mu K$ level. The spurious signals due to pointing error, differential beamwidth and beam ellipticity all peak at angular scales comparable to the beam size. If the beam size is $\approx 1^\circ$ the beam systematics mainly affect the deduced tensor-to-scalar ratio, r. If the polarimeter's beamwidth is a few arcminutes the associated systematics will impact the measured neutrino mass m_ν, spatial curvature Ω_k, running of the scalar spectral index α and the dark energy equation of state w (which strongly affects the lensing-induced B-mode signal). Ideally, the beamwidth should be as narrow as possible, preferably narrow enough as to push any systematics beyond the science range of $l < 2000$ or 3000. It can certainly be the case that other cosmological parameters will be affected as well. Two other spurious polarization signals we explore are due to differential gain and beam rotation; these effects are associated with different beam 'normalizations' and orientation, respectively, and are independent of the coupling between beam substructure and the underlying temperature perturbations. In particular, they have the same l-dependence as the primordial temperature anisotropy and polarization power spectra, respectively, and their maximum impact will be on scales associated with the CMB's temperature anisotropy ($\approx 1^\circ$) and polarization ($\approx 10'$). We illustrate several systematics assuming the underlying sky is an unpolarized point source in Figure 1. Figure 2 focuses on the effect of ellipticity and illustrates how differential ellipticity induces spurious polarization from an unpolarized underlying sky.

Since in real space the temperature and polarization patterns are convolved with the beams, these expressions are simply the product of their Fourier transforms in Fourier space. We restrict the discussion to an elliptical gaussian beam (with major

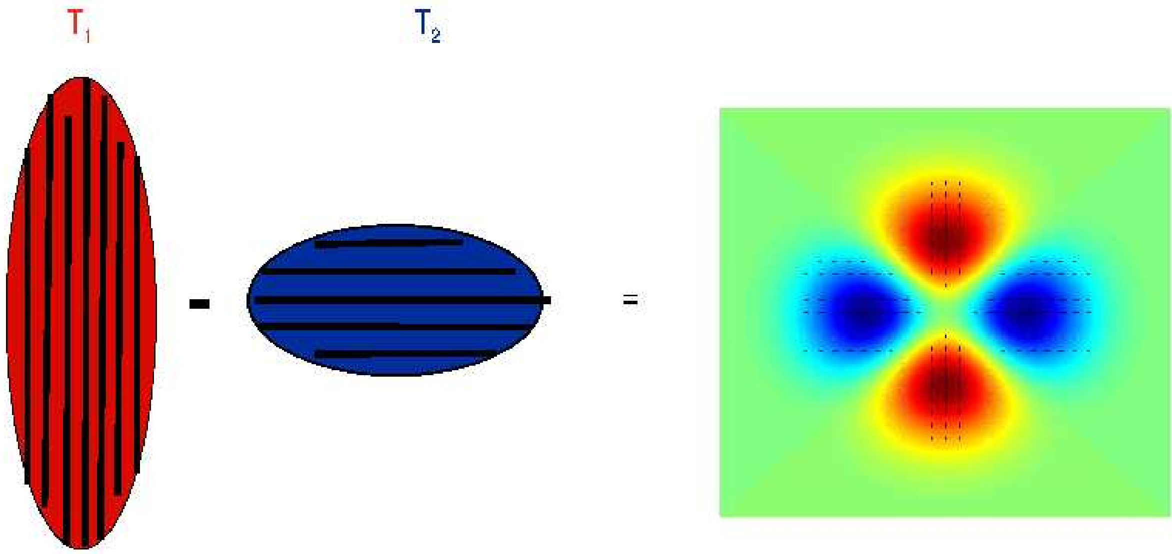

Figure 2. Spurious polarization induced by differential ellipticity: In this case, even though the two beams have the same ratio of principal axes in absolute value, one of the ellipticities is positive while the other is negative - this differential ellipticity results in non-vanishing polarization. The straight black lines represent the polarization directions of the two bolometers which is this case will induce only E-mode polarization.

and minor axes σ_x and σ_y and pointing represented by ρ_x and ρ_y)

$$B(\mathbf{x}) = \frac{1}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{(x-\rho_x)^2}{2\sigma_x^2} - \frac{(y-\rho_y)^2}{2\sigma_y^2}\right). \quad (1)$$

Its Fourier transform is

$$\tilde{B}(\mathbf{l}) = \exp\left(-\frac{l_x^2\sigma_x^2}{2} - \frac{l_y^2\sigma_y^2}{2} + i\mathbf{l}\cdot\rho\right). \quad (2)$$

The pointing error merely shifts the phase of the beam representation in Fourier-space. It is useful to switch to polar coordinates at this point where we use angles ψ, α and θ which are defined below. The Fourier representation of the beam (Eq.2) then becomes

$$\tilde{B}(\mathbf{l})d^2\mathbf{l} = e^{-y-z\cos 2(\phi_l+\psi-\alpha)+il\rho\cos(\phi_l-\alpha-\theta+\psi)}ldld\phi_l \quad (3)$$

where

$$\begin{aligned} y &\equiv \frac{l^2}{4}(\sigma_x^2+\sigma_y^2) \\ z &\equiv \frac{l^2}{4}(\sigma_x^2-\sigma_y^2) \approx (l\sigma)^2 e \end{aligned} \quad (4)$$

where in the last approximation we have made use in the definition of ellipticity $e \equiv \frac{\sigma_x-\sigma_y}{\sigma_x+\sigma_y}$ (Table 1). Employing the expansion of 2D plane waves in terms of cylindrical Bessel functions, Eq.(3) becomes

$$\begin{aligned} \tilde{B}(\mathbf{l}) &= e^{-y}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} i^{2m+n}I_m(z)J_n(l\rho) \\ &\times e^{i(2m+n)\psi-in\theta}e^{i(2m+n)(\phi_l-\alpha)} \\ &\equiv \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} B_{m,n}e^{i(2m+n)(\phi_l-\alpha)} \end{aligned} \quad (5)$$

where $\alpha \equiv \beta+\theta+\psi$ is the angle of the polarization axis in some coordinate system fixed to the sky (See angles layout in Figure 3). Using this basic result and the definition of optimal polarization maps (assuming white noise) the spurious power spectra involving E- and B-polarization modes can be compactly summarized in Tables 2 & 3 (Shimon et al. 2008). The systematics-induced power spectra are expressed in terms of the underlying CMB power spectra and certain functions of the relevant non-monopolar moments of scanning strategy. The small scale features of the spurious spectra (essentially

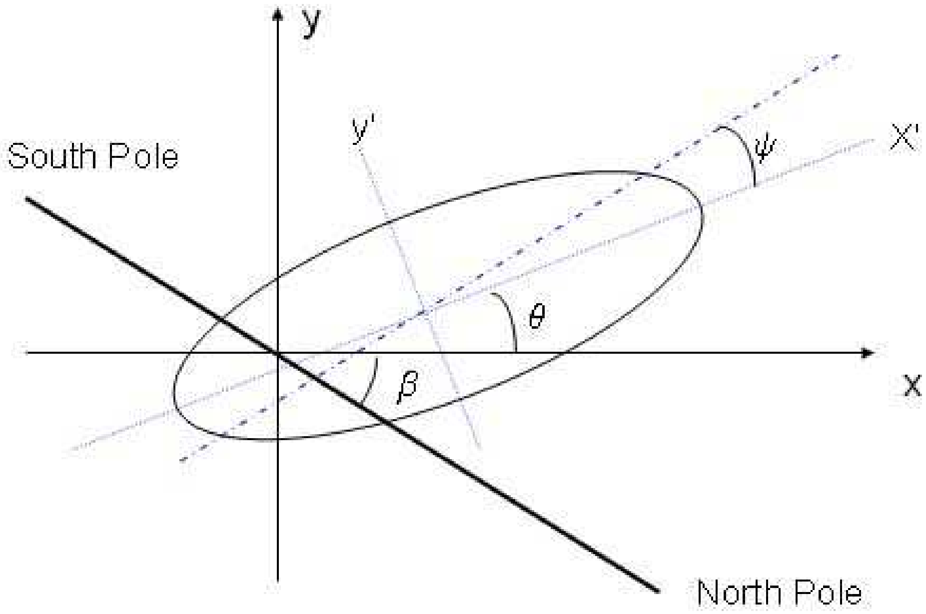

Figure 3. Angles layout for a single beam.

first and second derivatives of the temperature anisotropy in the underlying sky) can be obtained from the leading order Taylor expansions of the Cylinderical and Modified Bessel functions. Also, in some cases beam systematics couple to non-ideal moments of the scanning strategy (and vanish when the latter is ideal) - the relevant functions appear in convolutions with the underlying sky spectra. Finally, all these determine the level of leakage of temperature to polarization, but how this splits between the E- and B-modes is determined by the angles θ and ψ as described in Tables 2 & 3. This analytic calculation was generalized to arbitrary beam shapes in Miller, Shimon & Keating (2009a).

3 THE EFFECT OF SYSTEMATICS ON LENSING RECONSTRUCTION

Gravitational lensing of the CMB is both a nuisance and a valuable cosmological tool (e.g. Zaldarriaga & Seljak 1998). It certainly has the potential to complicate CMB data analysis due to the non-gaussianity it induces. However, it is also a unique probe of the growth of structure in the linear, and mildly non-linear, regimes (redshift of a few). Kaplinghat, Knox & Song (2003), Lesgourgues et al. (2006), as well as others, have shown that with a *nearly ideal CMB experiment*, neutrino mass limits can be improved by a factor of approximately four by including lensing extraction in the data analysis using CMB data alone. This lensing extraction procedure is not perfect; a fundamental residual noise will afflict any experiment, even ideal ones. This noise will, in principle, propagate to the inferred cosmological parameters if the latter significantly depend on lensing extraction, e.g. neutrino mass, α (running of the scalar index) and w (dark energy equation of state). By optimally filtering the temperature and polarization Hu & Okamoto (2002) reconstructed the lensing potential from quadratic estimators. It was shown that for experiments with ten times higher sensitivity than Planck, the EB estimator yields the tightest limits on the lensing potential. This result assumes no beam systematics which might significantly contaminate the observed B-mode. That is, it is certainly possible that although for high sensitivity *systematics-free* experiments the EB estimator yields the best lensing reconstruction it will be the TT estimator that will best construct the lesing potential in practice since it is virtually insensitive to the nearly negligible beam systematics. It is true however, as will be discussed in section 5, that higher sensitivity experiments are expected to have lower *absolute* systematics (which are suppressed by $(S/N)^{-2}$ where S is the bright signal used for beam calibration and N is the effective instrumental noise). Larger S/N implies both better EB-reconstruction where at the same time insures lower systematics.

Accounting for beam systematics in both Stokes parameters and lensing power spectra is straightforward. In addition, the instrumental noise associated with the main beam is accounted for, as is conventional, by adding an exponential noise term. Assuming gaussian white noise

$$N_l = \frac{1}{\sum_a (N_l^{aa})^{-1}} \tag{6}$$

where a runs over the experiment's frequency bands. The noise in channel a is (assuming a gaussian beam)

$$N_l^{aa} = (\theta_a \Delta_a)^2 e^{l(l+1)\theta_a^2/8\ln(2)}, \tag{7}$$

where Δ_a is the noise per pixel in μK-arcmin, θ_a is the beam width, and we assume noise from different channels is uncorrelated. The *observed* power spectrum then becomes

$$C_l^X \rightarrow C_l^X + N_l^X \tag{8}$$

where X is either the auto-correlations TT, EE and BB or the cross- correlations TE, TB and EB [the latter two power spectra vanish in the standard model but not in the presence of beam systematics and exotic parity-violating physics (section 4 below) e.g. Carroll (1998), Liu, Lee & Ng (2006), Xia et al. (2008), Komatsu et al. (2008) or primordial magnetic fields e.g. Kosowsky & Loeb (1996) and see Section 4]. For the cross-correlations, the N_l^X vanish as there is no correlation between the instrumental noise of the temperature and polarization (in the absence of beam systematics). The quadratic estimators method utilizes both theoretical and observed power spectra. Beam systematics merely serve to increase the variance of the latter

$$C_l^{X,obs} \rightarrow C_l^{X,obs} + C_l^{X,sys} \tag{9}$$

The effect of differential gain, beamwidth, beam rotation, ellipticity and differential pointing on the noise of lensing reconstruction was illustrated (Smith et al. 2008) with 20 different combinations of noise levels and resolution for CMBPOL spanning the sensitivity and resolution ranges 1-6 μK $-$ arcmin and 3-30 arcmin, respectively. It was found that the most stringent constraints on allowed levels of beam systematics come from the inflationary signal which seems to be more permeable to B-mode systematics and the lensing-dependent cosmological parameters are less prone to B-mode systematics. Especially relevant to lensing inference is the residual noise of recovered deflection angle N_l^{dd}. The modified noise in reconstructing the deflection angle, N_l^{dd}, is consistently substituted into the Fisher-matrix and propagated through the parameter inference procedure (Miller, Shimon & Keating 2009a).

4 BEAM SYSTEMATICS AND 'COSMIC BIREFRINGENCE'

CMB's reputation as a unique *cosmological* tool due to the underlying linear physics on a wide range of scales reflects only one aspect of this invaluable cosmological probe. A photon traveling over time scales of $\sim$ 10 Gyr experiences very long 'integration time', literally. In some cases, minuscule effects, which cannot be detected in terrestrial experiments such as particle accelerators, optical/microwave resonators, torsion pendulums, etc., accrue to observable levels in the CMB anisotropy and polarization.

Lorentz-violating and/or parity-violating terms in the electromagnetic (EM) or gravitational sectors of a hypothetical unified model of the fundamental interactions of nature (e.g. Carroll, Field & Jackiew 1990) may have far-reaching implications on propagation of polarized light. We know for a fact that the weak interaction exhibits a small degree of parity-violation, and if all four fundamental interactions of nature are to be unified at high energies, we expect some degree of parity-violation in the other interactions as well. Moreover, the most promising candidate for a 'theory of everything', string theory, or higher dimensional gravitation theories abound with parity-violating terms (e.g. Das, Jain & Mukherji 2001, Bezerra, Ferreira, & Helayël-Neto 2005, Maity, Roy & Sengupta 2008). The fact that the effective low-energy standard model of particle physics works extremely well in explaining a host of phenomena does not entirely exclude the possibility that either Lorentz- and/or parity-invariance are spontaneously broken symmetries of the underlying fundamental unified theory. Lorentz-violating (but parity-even) coupling of the EM radiation to matter fields exists in few field-theoretic scenarios (Wolf 2001), e.g. axion interaction with EM radiation in the presence of magnetic field (Sikivie 1983), EM interaction with torsion in non-metric theories of gravitation (Wolf 1999), coupling to scalar fields (Lue, Wang & Kamionkowski 1999), quantum gravitational effects (Wolf 2001), etc.

The observational efforts to detect these (possibly tiny) effects range from earth based optical and microwave resonant-cavities and search for fractional changes in the resonant frequency (e.g. Brillet & Hall 1979), to astrophysical observations of time-lags in neutron star pulsations, rotation of the polarization plane from observations of polarized radio-sources (Carroll & Field 1997) to parity-violating correlations in the CMB power spectra (Feng et al. 2006, Liu, Lee, & Ng 2006, Cabella, Natoli, & Silk 2007, Xia, Li, Wang & Zhang 2008, Komatsu et al. 2008 and Wu et al. 2008)

Terms of the $F_{\mu\nu}\tilde{F}^{\mu\nu}$ type ($F_{\mu\nu}$ is the EM field-strength and $\tilde{F}^{\mu\nu}$ is its dual) can still be added to the Lagrangian of the photon sector in the EM Lagrangian and preserve parity symmetry (if coupled to another pseudo-scalar field). However, in this case Lorentz invariance is broken and the photon acquires an effective mass. Current *upper limits* on the photon mass are very stringent; these Lorentz- and parity-breaking effects may be extremely small. The CMB is an ideal probe of Lorentz- and parity-violating interactions, because it provides us with the longest optical path length to detect coupling of scalar fields to EM (several models which predict a small level of Lorentz-violating models have the property that most of the polarization rotation occurs at high redshifts e.g. Finelli & Galaverni 2008) and supplements other astrophysical probes such as the rotation of polarization plane of radio sources. This phenomena is collectively referred to as 'cosmic birefringence' (CB).

Current constraints on CB come from the observed TT, EE and TE correlations. the CMB temperature anisotropy is insensitive to the rotation angle γ of the polarization plane but E-mode polarization is (since rotation of the polarization plane rotates the Q and U Stokes parameters resulting in mixing of E and B and vice versa). All upper limits on the rotation angle to date essentially come from comparing the observed EE and TE (based on TT observations) to the expected EE and TE correlations assuming the standard cosmological model (both TT and EE are induced by the same mechanism - primordial scalar (density) perturbations). A small deficit in TE and EE compared to what is naively expected based on the observed TT auto-correlation - may be attributed to E-leakage to B by CB. Clearly, while a small leakage from EE may indeed result in a small fractional change, such a rotation is expected to have a relatively much larger impact on the (far smaller) BB correlations. The latter is therefore an indispensable tool in constraining this non-standard physics. The EB and TB are naively expected to do even better (based on S/N considerations). These correlations should vanish in the absence of a preferred chirality-direction, e.g. in standard cosmology and much the same way that a detection of BB at $l \approx 1000$ will signal a clear detection of CMB lensing an unambiguous TB and EB detection will be a smoking gun for CB (or beam systematics).

Any rotation of the polarization-plane by an angle γ, either of cosmological origin (e.g. Feng et al. 2006, Liu, Lee, & Ng 2006, Cabella, Natoli, & Silk 2007, Xia, Li, Wang & Zhang 2008, Komatsu et al. 2008) or otherwise, will mix the E and B modes

$$\begin{aligned} E' &= E\cos 2\gamma - B\sin 2\gamma \\ B' &= E\sin 2\gamma + B\cos 2\gamma \end{aligned} \tag{10}$$

and as a result

$$\begin{aligned} C_l'^{TE} &= C_l^{TE}\cos 2\gamma \\ C_l'^{TB} &= C_l^{TE}\sin 2\gamma \\ C_l'^{EE} &= C_l^{EE}\cos^2 2\gamma + C_l^{BB}\sin^2 2\gamma \\ C_l'^{BB} &= C_l^{EE}\sin^2 2\gamma + C_l^{BB}\cos^2 2\gamma \\ C_l'^{EB} &= \frac{1}{2}(C_l^{EE} - C_l^{BB})\sin 4\gamma. \end{aligned} \tag{11}$$

Clearly, C_l^{TT} does not participate in this mixing. However, beam systematics can leak (preferentially) C_l^{TT} and C_l^{TE} (instrumental-polarization) to C_l^{TB} and C_l^{EB} as described in Table 3. While the effect of cross-polarization is trivial to assess since it results in a pure bias $\gamma \rightarrow \gamma + \varepsilon$, the other effects we consider in Table 3 are likely to bias the inferred cosmological γ but in a more contrived manner and a full simulation is needed in this case (Miller, Shimon & Keating 2009b). Also, the leakage from the TT sector due to beam systematics, can be potentially more dangerous to the BB correlations than are the cosmological rotation and cross-polarization (that leak from EE only).

5 BEAM CALIBRATION AND ESTIMATES OF SYSTEMATIC NOISE

Until now we described the manifestation of beam systematics in the power spectra in a parametric form, i.e. we expressed the spurious C_l^B, C_l^{TB} etc. in terms of the underlying power spectra and the beam parameters, such as ellipticity e, gain g, beamwidth μ, etc (Tables 2 and 3). Now, given the number of detectors, their NET, integration time, area of sky covered and experiment optics, we ask what is the actual expected level of systematics ? That is, our ultimate goal is presenting the uncertainty in beam parameters (and therefore the systematics-induced power spectra) in terms of the the S/N with which we can calibrate our beams - these are directly related to the expected level of *uncertainty* in beam parameters. The larger S/N we have, the smaller is the beam uncertainty and the spurious power spectra which derive from it. This is the classical problem of deconvolution in the presence of noise: *Wiener Filtering*. In this case we are interested in deconvolving the beam image from a point-like window function - the calibration point-source (Jupiter). By optimally filtering a noisy image we obtain the resulting S/N

$$\left(\frac{S}{N}\right)^2 = \int \frac{|\tilde{S}(\mathbf{l})|^2}{P(\mathbf{l})}\frac{d^2\mathbf{l}}{(2\pi)^2}, \tag{12}$$

where $\tilde{S}$ is the Fourier transform of the *observed* temperature and $P(l)$ is the detector's white noise power spectrum $(\Delta_b\theta_b)^2$ (where Δ_b is NET and θ_b is the beam FWHM). The ratio de-weights modes with large noise. In our case, however, the detector noise is assumed white and all modes are equally suppressed. Assuming a 'point'-source with temperature profile

$$T_p = Af\left(\frac{\theta}{\theta_p}\right) \tag{13}$$

where f is a function of the ratio $\frac{\theta}{\theta_p}$ and θ_p is the typical size of the source (Jupiter subtends 0.5 arcmin on the sky and compared to the beams of most CMB polarimeters it is well-approximated by a point source). The Fourier transform of the source T_p with the beam reads

$$\tilde{T}_p^{\rm obs} = (1+g)\tilde{T}_p e^{-\frac{1}{2}l_x^2\sigma_x^2-\frac{1}{2}l_y^2\sigma_y^2-i\mathbf{l}\cdot\rho} \tag{14}$$

where we assume an elliptical gaussian beam with pointing ρ, major and minor axes σ_x and σ_y and calibration error g. Since the pointing error induces only a phase shift in the beam function, $|\tilde{T}_p^{\rm obs}|$ is completely insensitive to it. Here $\Delta_b\theta_b$ is given in μK-arcmin units and θ_b is the average beamwidth. The relation between σ_x, σ_y, θ_b and ellipticity is

$$\begin{aligned}\sigma_x &= \theta_b(1+e)/(\sqrt{8\ln 2})\\ \sigma_y &= \theta_b(1-e)/(\sqrt{8\ln 2}).\end{aligned} \tag{15}$$

The Fourier transform of Jupiter's temperature reads

$$\tilde{T}_p = A\int f\left(\frac{\theta}{\theta_p}\right)e^{i\mathbf{l}\cdot\theta}d^2\theta = 2\pi A\int_0^\infty f\left(\frac{\theta}{\theta_p}\right)J_0(l\theta)\theta d\theta = 2\pi A\theta_p^2\int_0^\infty f(x)J_0(l_px)xdx \tag{16}$$

where we have used the expansion of 2D plane-waves in terms of cylindrical Bessel functions

$$e^{i\mathbf{l}\cdot\theta} = \sum_{n=-\infty}^{\infty} i^n J_n(l\theta)e^{in(\phi_l-\phi)} \tag{17}$$

and defined $l_p \equiv l\theta_p$. If we now replace $f(x)\to\delta(x)/x$ we obtain

$$\tilde{T}_p = 2\pi A\theta_p^2. \tag{18}$$

Carrying out the integration over the beam function (Eq. 15), we obtain

$$\left(\frac{S}{N}\right)^2 = \frac{2\ln(2)(1+g)^2}{\pi(1-e^2)}\left(\frac{\theta_p}{\theta_b}\right)^4\left(\frac{T_p}{\Delta_b}\right)^2\eta^2 f_t f_{sky} \tag{19}$$

i.e. the S/N scales as the filling factor of the source to the beam, as expected from the point-source-approximation. The S/N is also proportional to T_p/Δ_b (Δ_b is the NET) as expected and is suppressed by $\eta\sqrt{f_t f_{sky}}$ where η is the optical efficiency

Now, the uncertainty in ellipticity is derived by the following argument. We can vary the ellipticity and thereby change the signal but as long as ΔS is small compared to the noise N there is no way to distinguish the two; the obstacle in precisely determining the beam's ellipticity (and thereby correct for it) is the instrumental noise.

By requiring that the change in the beam parameters will induce a change in the signal equal to the noise, we find the expected uncertainty in ellipticity, beamwidth and gain. Assuming no gain and beamwidth uncertainty and letting the ellipticity vary we obtain

$$\begin{aligned}(S/N)^2 &= \frac{const.}{\theta_b^2}\\ (S'/N)^2 &= (S/N)^2+1 = \frac{const.}{\theta_b^2(1-e^2)}\end{aligned} \tag{20}$$

and therefore

$$\Delta(e^2) = (N/S)^2. \tag{21}$$

We can repeat the same exercise for the gain and beamwidth. This procedure allows us to express the systematics in terms of S/N and the underlying anisotropy power spectrum (compare Table 2)

$$\begin{aligned}C_l^{B,gain} &= 2(N/S)^2C_l^T\\ C_l^{B,mono} &= C_l^{B,ellip}\approx 2(N/S)^2(l\sigma)^4C_l^T\end{aligned} \tag{22}$$

The pre-factor 2 accounts for the fact that we consider differences between beams which will increase the uncertainty by a factor $\sqrt{2}$ and the power spectra, therefore, by a factor 2 (note that in Shimon et al. 2008 we assumed in several cases that one of the two beams is systematics-free and the current treatment is therefore more conservative). Also, we assume 'worst-case-scenario' as far as scanning strategy (in the cases of differential beamwidth and gain) as well as the angle $\psi = 45^\circ$ (in the case of ellipticity) are concerned. Δ_b are the source temperature (≈ 150K for Jupiter) and NET of the instrument (see Smith et al. 2008 for the experiment parameters considered). We assume fraction sky coverage of $f_{sky} = 0.65$. Typical N/S levels we obtain are sufficiently small ($\sim 10^{-5}-10^{-4}$) to insure negligible effect on the inflationary and lensing science (Smith et al. 2008) at least for the differential gain, beamwidth and ellipticity (see Table 4). The two other beam systematics, beam rotation and differential pointing, add only a phase and are therefore absent from the numerator of Eq.(12). They, therefore, cannot be constrained by this method.

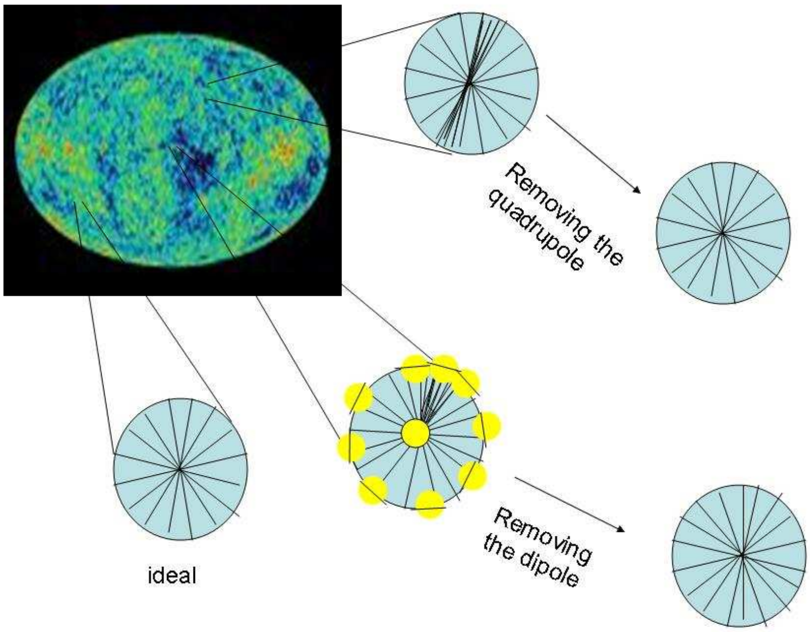

Figure 4. An illustration of how a non-ideal scanning strategy that contains a non-vanishing quadrupole moment in scanning strategy, which conspires with the differential beamwidth and gain to give rise to spurious B-mode, can be remedied by removing those 'hits' which contribute to the quadrupole. Similarly, the dipole of the scanning strategy couples to gradients in the underlying temperature anisotropy as well as to the telescope pointing to generate spurious polarization. Removing data points which contribute to this dipole will suppress this systematic. The lines represent the polarization angle coverage and in this plot the quadrupole moment is apparent.

6 MITIGATION: 'IDEALIZING' THE SCANNING STRATEGY

As several of the beam systematics considered here couple beam-imperfections to the scanning strategy it can be advantageous to devise techniques to 'idealize' the scanning strategy after data acquisition is complete. An illustration of quadrupole-removal from the scanning strategy is depicted in Figure 4. In the case of circularly-symmetric monopolar beam imperfections, such as differential gain and differential beamwidth, a vanishing quadrupole moment of scanning strategy is expected to result in negligible systematics. Similarly, the differential pointing effect couples to the dipole and octupole moments of scanning strategy and if the latter is removed from our data the first order pointing, a reducible effect in this sense, vanishes and only the second order, negligibly small, pointing effect survives. This idea, naturally, requires extensive simulations for realistic experiments and this is under investigation. While this simple idea has never been tested, we feel that it can be used to alleviate at least one of the effects not considered in the preceding section: the effect of differential poining, which incidentally proves to be one of the most pernicious of beam systematics in several ongoing CMB polarization experiments. Beam rotation, not addressed until now in this summary in terms of its expected level and how it can be mitigated is a genuinly distinct effect because it is not a function of beam-calibration nor does it couple to non-ideal scanning strategy. It seems, at this point, that the only way to minimize this effect is to better measure the rotation angle but this may prove to be extremely challeging, depending on the tolerance level on cosmological parameters (Smith et al. 2008).

7 SUMMARY

The ultimate criterion for tolerance levels of beam systematics is how they propagate and affect cosmological parameter estimation. O'Dea, Challinor & Johnson (2007) and Miller, Shimon & Keating (2009a, 2009b) considered the effect of both irreducible and reducible systematics. For reducible systematics the scanning strategy is a free parameter in our analysis (Miller, Shimon & Keating 2009a; under the assumption it is non-ideal, yet uniform, over the map) and we set limits on the product of the scanning strategy (encapsulated by the f_1 and f_2 parameters) and the differential gain, beamwidth and pointing.

Our analysis was limited to the tensor-to-scalar ratio r, total neutrino mass M_ν, dark energy equation of state w, curvature Ω_k, and the tilt of the scalar index α. While r is mainly constrained by the primordial B-mode signal that peaks on degree scales (and is therefore not expected to be overwhelmed by the beam systematics which peak at sub-beam scales), it is still susceptible to the tail of these systematics, extending all the way to degree scales, because of its expected small amplitude (less than $0.1\mu K$). The tensor-to-scalar ratio is also affected by differential gain and rotation which are simply rescallings of temperature anisotropy and E-mode polarization power spectra, respectively, and therefore do not necessarily peak at scales beyond the primordial signal.

Ideally, the lensing signal, which peaks at $l \approx 1000$, provides a useful handle on the neutrino mass as well as other cosmological parameters which govern the evolution of the large scale structure and gravitational potentials. However, the inherent noise in the lensing reconstruction process (Hu & Okamoto 2002) which depends, among others, on the instrument specifications (instrumental noise and beamwidth), now depends on beam systematics as well. The systematics, however, depend on the cosmological parameters through temperature leakage to polarization, and as a result there is a complicated interplay between these signals and the information they provide on cosmological parameters. As our numerical calculations show, the effect on the inferred cosmological parameters stems from both the *direct effect* of the systematics on the parameters and the *indirect effect* on the noise in the lensing reconstruction, N_l^{dd}, in the particular cases that the minimum variance (MV) estimator (which largely builds on the EB correlations that are most sensitive to lensing).

The Fisher information-matrix gives a first order approximation to the lower bounds on errors inferred for these parameters. We followed O'Dea, Challinor, & Johnson (2007) in quantifying the required tolerance on the differential gain, differential beamwidth, pointing, ellipticity and rotation (Miller, Shimon & Keating 2009a). To estimate the effect of systematics and to set them to a given tolerance limit one has to compare the *systematics-free* 1σ error in the i-th parameter to the error obtained in the presence of systematics. The latter has two components; the bias and the uncertainty. The latter depends on the curvature of the likelihood function, i.e. to what extent does the information matrix constrain the cosmological model in question. As in O'Dea, Challinor, & Johnson (2007) and Miller, Shimon & Keating (2009a) we define

$$\begin{aligned} \delta &= \frac{\Delta\lambda_i}{\sigma_{\lambda_i}}|_{\lambda_i^0} \\ \beta &= \frac{\Delta\sigma_{\lambda_i}}{\sigma_{\lambda_i}}|_{\lambda_i^0} \end{aligned} \tag{23}$$

where the superscript 0 refers to values evaluated at the peak of the likelihood function, i.e. the values we assume for the underlying model, and $\Delta\lambda_i$ and $\Delta\sigma_{\lambda_i}$ are the bias and the change in the statistical error for a given experiment and for the parameters λ_i induced by the beam systematics, respectively. As shown in O'Dea, Challinor, & Johnson (2007) these two parameters depend solely on the primordial, lensing and systematics power spectra. We require that neither δ nor β exceed 10% of the uncertainty without systematics. If the systematics-induced error is independent of the statistical error, the two effects added in quadratures are $\sigma_\lambda \rightarrow \sigma_\lambda\sqrt{1+\delta_\lambda^2} \approx \sigma_\lambda(1+\frac{1}{2}\delta_\lambda^2)$ which is smaller than 1% but if they are fully-correlated $\sigma_\lambda \rightarrow \sigma_\lambda(1+\delta_\lambda)$, resulting in a 10% increase in the error; again a small impact.

The upper limits we obtain on the allowed range of beam mismatch parameters for given experiments give arbitrarily-set tolerance levels on the parameter bias and uncertainty and therefore constitute very conservative limits. It can certainly be the case that some of the systematics studied here may be fully or partially removed. This includes, in particular, the first-order pointing error which couples to the dipole moment of non-ideal scanning strategies (see Shimon et al. 2008). By removing this dipole during data analysis the effect due to the systematic first order pointing error (dipole) drops dramatically. Our results highlight the need for scan mitigation techniques because the coupling of several beam systematics to non-ideal scanning strategies result in systematic errors. A brute-force strategy to idealize the data could be to remove data points that contribute to higher-than-the-monopole moments in the scanning strategy. This would effectively make the scanning strategy 'ideal' and alleviate the effect of the *a priori* most pernicious beam systematics. This procedure 'costs' only a minor increase in the instrumental noise (due to throwing out a fraction of the data) but will greatly reduce the most pernicious reducible beam systematic, i.e. the first order pointing error ('dipole' effect).

CMB polarization, specifically the B-mode polarization, is expected to open a new window to a possible detection of non-standard physics through TB and EB correlations. Beam systematics, however, can contaminate these measurements and bias the inferred cosmological parameters. CMB observations, especially those which will have the required sensitivity and fidelity to detect the 'forbidden' power spectra C^{TB} and C^{EB}, will be able to set tight constraints on cosmic birefringence, a phenomena related to the bedrock of fundamental physics and its symmetries. Cosmology improves on terrestrial experiments in this context due to the large optical path lengths which enable the detection, in principle, of extremely small effects. However, since a statistically significant detection of CB heavily relies on measuring the TB and EB 'smoking gun' correlations, and due to the fact that these involve the sub-μK B-mode signal which is prone to numerous systematics, a credible detection of non-vanishing γ should account for these possible sources of confusion. We saw that beam-rotation, differential ellipticity, as well as differential pointing, gain and beamwidth (to a lesser extent) can bias the inferred γ and also change the uncertainty (in the case of differential pointing and ellipticity) due to their direct effect on the TB and EB correlations. While the differential pointing is very small due to its weak l-dependence (compared to other systematics considered here) and the effect of beam ellipticity can be partially harnessed (Appendix A), it is the beam rotation effect which mainly contaminates CB. In particular, we have shown in Miller, Shimon & Keating (2009b) that CMB experiments which are optimized for inflationary and lensing

depends on beam substructure	effect	parameter	definition
No	gain	g	$g_1 - g_2$
Yes	monopole	μ	$\frac{\sigma_1-\sigma_2}{\sigma_1+\sigma_2}$
Yes	dipole	ρ	$\rho_1 - \rho_2$
Yes	quadrupole	e	$\frac{\sigma_x-\sigma_y}{\sigma_x+\sigma_y}$
No	rotation	ε	$\frac{1}{2}(\varepsilon_1 + \varepsilon_2)$

Table 1. Definitions of the parameters associated with the systematic effects. Subscripts 1 and 2 refer to the first and second polarized beams of the dual beam polarimeter assumed in this work.

effect	parameter	ΔC_l^{TE}	ΔC_l^E	ΔC_l^B
gain	g	0	$g^2 f_1 \star C_l^T$	$g^2 f_1 \star C_l^T$
monopole	μ	0	$4\mu^2(l\sigma)^4 C_l^T \star f_1$	$4\mu^2(l\sigma)^4 C_l^T \star f_1$
pointing	ρ	$-c_\theta J_1^2(l\rho) C_l^T \star f_3$	$J_1^2(l\rho) C_l^T \star f_2$	$J_1^2(l\rho) C_l^T \star f_2$
quadrupole	e	$-I_0(z) I_1(z) c_\psi C_l^T$	$I_1^2(z) c_\psi^2 C_l^T$	$I_1^2(z) s_\psi^2 C_l^T$
rotation	ε	0	$4\varepsilon^2 C_l^B$	$4\varepsilon^2 C_l^E$

Table 2. The scaling laws for the systematic effects to the power spectra C_l^T, C_l^{TE}, C_l^E and C_l^B assuming the underlying sky is not polarized (except for the *rotation* signal where we assume the E, and B-mode signals are present) and a general, not necessarily ideal or uniform, scanning strategy. The next order contribution (10% of the 'pure' temperature leakage shown in the table) is contributed by C_l^{TE}. It can be easily calculated based on the general expressions in Shimon et al. (2008) where the definitions of z, ρ, ε, etc., are also found. For the pointing error we found that the 'irreducible' contribution to B-mode contamination, arising from a second order effect, is extremely small and therefore only the first order terms (which vanish in ideal scanning strategy) are shown. The functions f_1 and f_2 are experiment-specific and encapsulate the information about the scanning strategy which couples to the beam mismatch parameters to generate spurious polarization. In general, the functions f_1 and f_2 are spatially-anisotropic but for simplicity, and to obtain a first-order approximation, we consider them constants. In the case of ideal scanning strategy they identically vanish. The exact expressions are given in Shimon et al. (2008).

science *may not* be adequate for the CB science that requires a much better control of beam rotation. The nominal γ-detection with PLANCK and POLARBEAR is 3.8 and 1.1 arcminutes, respectively. Beam rotation should therefore be controlled to 0.75 and 0.21 arcminutes if a 5σ detection of CB is required. This systematic effect cannot be mitigated by scanning strategy. The effect of differential pointing (which is subdominant to the effect of differential ellipticity) can be further suppressed by scanning strategy mitigation. The differential-ellipticity, together with the beam rotation effect, seem the most pernicious for TB spectra.

As was shown in Miller, Shimon & Keating (2009b), the minimum requirement will in general not suffice for a credible CB detection via the 'forbidden' TB and EB correlations. The likely reason for this is that while BB correlations are quadratic in the beam-imperfection parameters (e.g. $\propto e^2, \varepsilon^2$) the TB correlations are only linear in these *small* beam parameters (and the EB correlation is in general noisy). This implies that for a given ellipticity or beam rotation level the fractional bias in the TB cross-correlations will be larger than the the corresponding fractional bias in the BB power spectrum by O(1/e) and O(1/ε), respectively. Our conclusion regarding the CB science is that while TB and EB are unique indicators for new-physics in principle, they can in practice be excited by imperfect beams and in order to realize the promising potential of the high-sensitivity and fine-resolution PLANCK and POLARBEAR at CB detection beam rotation has to be controlled to the sub-arcminute level. Unless beam rotation is controlled to the arcminute level, a conservative approach, which does not use C_l^{TB} and C_l^{EB}, may be more adequate for the CB science.

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effect	parameter	ΔC_l^{TB}	ΔC_l^{EB}
gain	g	0	0
monopole	μ	0	0
pointing	ρ	$\frac{1}{2}J_2(l\rho)[1+J_0(l\rho)]s_\theta C_l^T$ $+s_\theta J_1^2(l\rho)C_l^T\star f_3$	$s_\theta c_\theta J_2^2(l\rho)C_l^T$
quadrupole	e	$-I_0(z)I_1(z)s_\psi C_l^T$	$I_1^2(z)s_\psi c_\psi C_l^T$
rotation	ε	$2\varepsilon C_l^{TE}$	$2\varepsilon C_l^E$

Table 3. The contribution of the systematic effects to the power spectra C_l^{TB}, C_l^{EB} assuming the underlying sky is not polarized (except for the *rotation* signal when we assume E-, and B-mode polarization are present) and general sky scanning.

	$1\mu K'$	$2\mu K'$	$4\mu K'$	$6\mu K'$
3'	1.64×10^{-5}	3.27×10^{-5}	6.55×10^{-5}	9.82×10^{-5}
5'	2.73×10^{-5}	5.46×10^{-5}	1.09×10^{-4}	1.64×10^{-4}
10'	5.46×10^{-5}	1.09×10^{-4}	2.18×10^{-4}	3.27×10^{-4}
20'	1.09×10^{-4}	2.18×10^{-4}	4.36×10^{-4}	6.55×10^{-4}
30'	1.64×10^{-4}	3.27×10^{-4}	6.55×10^{-4}	9.82×10^{-4}

Table 4. N/S for a range of sensitivities and angular resolution experiments using the calibration procedure outlined in section 5. We assume $\eta=5\%$, $f_t=1/30$ and $f_{sky}=0.65$ (somewhat different values from those used in Smith et al. 2008). The N/S scales as $\eta^{-1}(f_t\cdot f_{sky})^{-1/2}$.

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